Generalized Set Analysis of African Music
A Critique and Expansion of the Theories of Willie Anku

Michael Scott Cuthbert
Harvard University

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In a series of pamphlets and articles published from 1992 to 2000, Willie Anku set forth theories of organization in African rhythm, primarily focusing on recurring patterns in drumming. In these publications, Anku joined theorists such as Jay Rahn and Jeff Pressing in postulating the applicability of set analysis and group theory in the study of African music, and, more specifically, that tools originally designed to aid in the description and interpretation of pitch structures could be employed in exploring what they see as an isomorphic structure: beat classes in African drum music.

Anku published several theories of African rhythm which he believed could be applied cross-culturally to most music in sub-Saharan Africa. Anku based his most important analytical tool, RTP, on the relationship between the independent lines played by individual performers and a fundamental and recurring pattern of fixed length. Anku demonstrated how the position of phrases with respect to this base pattern varies throughout a work and how some of these phrases could be viewed as rotations of a generalized "prime form" of the phrase.

Anku's theories have been simultaneously well-received and yet sparsely applied.[1] Reflection on their implications has been near non-existent. Anku himself has only used his set analysis on two major works, an Adowa and a Bawa dance, which suggests that application of these theories could be a difficult labor. Unfortunately for those who want to follow in his footsteps, Anku has not documented procedures by which others can reproduce his analyses.

[†] I wish to thank Kofi Agawu in particular for supporting this work and for his highly astute comments on an earlier draft. David Kaminsky generously provided me with an unpublished transcription of Adowa to draw on. Meki Nzewi has kindly provided general advice on this project. I owe a tremendous debt of gratitude to Willie Anku for his words of support and interest in this work. [December 2003:] I have taken this opportunity to correct some typographical errors from the May 2000 version of this paper and to better integrate diagrams into the body of the text. Substantive changes are marked [*].

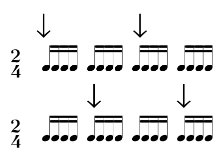
[1] Anku's 2000 article in *Music Theory Online* elicited a lively discussion in the journal's email discussion forum. I will return to this later.

This paper explores possible assumptions and methodologies which seem to be inherent in Anku's theories in order to begin to analyze his findings. I focus on three principles of drum rhythm which Anku believes can be found in 12 or 16 beat sets[2] and which are relevant generally in sub-Saharan African music: regulative time points (RTPs), classification of rhythmic sets by prime forms, and the frequency of attacks as a measure of the intensity level of a work. Although Anku believes these principles to apply broadly, I have chosen for the sake of space to narrow my examination to one repertory he has already studied, the *atumpan* line of Akan Adowa music, though I will occasionally examine the Dagaaba Bawa group of dances when they contradict or expand upon the Adowa examples.

Theory of Regulative Time Points

The most pervasive of Anku's theories has been that of the regulative time point (RTP). Appearing in all of his publications since his 1988 dissertation, RTP is simultaneously a new term for the strong pulse or downbeat of a recurring rhythmic line (called a timeline) and also a measure of the rhythmic relationship of any musical line to that strong pulse.[3] The RTP value for a line indicates on which pulse of the line (eighth notes in $\frac{12}{8}$, sixteenth notes in $\frac{2}{4}$) the strong pulse of the timeline falls:[4]

RTP 1:
RTP 5:



The arrows (↓) indicate felt or imagined downbeats. Labeling the RTP on the preceding example does not present a problem because the example occurs in isolation. Determining the begin-

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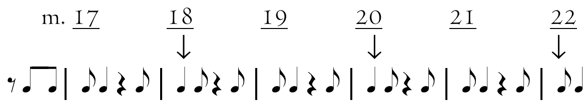
[2] Anku uses "beat" where many other writers use pulse, such as "twelve-beat set." From here on, I will adopt Anku's terminology. He also uses beat to mean a three pulse unit in Adowa (Anku, *Adowa*, p. 6).

[3] Anku has also referred to RTP as the "regulative pulse." I cannot confirm whether RTP was used in Anku's 1986 M.A. Thesis, "Rhythmic Procedures in Akan Adowa Drumming."

[4] In order to avoid confusion, I will endevor to use RTP only in the second sense, as a distance measured with respect to the strong beat of the timeline. Note that the timeline does not have to actually be played at any given moment in order to be regulative. The timeline can be mentally felt, either on the basis of having been played previously in the piece or as a part of the style of the genre. In this way, it is similar to the ways in which silent strong beats can still be felt in Western compositions.

ning or end of a 12 or 16 beat set is much more difficult when it occurs in a much larger context, as for example in this unlabeled section of Anku's 1997 article in the *Black Music Research Journal*:[5]



In this example, a new pattern begins two beats before m. 17 and the strong beat of the timeline falls on the ninth beat of the line, so it is RTP 9. By the end of m. 18, though, the pattern is beginning one pulse later, so we should label the second repetition as RTP 8. We should note that from the end of m. 17 on, RTP 2 also appears as a possibility; we cannot decide between these two interpretations. The pattern repeats until m. 28 when finally RTP 2 appears as more likely:[6]

This simple example is quite ambiguous. In the most difficult sections of the master drum parts, Anku's theoretical framework does not give us enough information to decide between interpretations.

Observations relating to RTP

Anku has never published his principles for determining the RTP orientation of a span. Since an understanding of how he arrives at his set groupings is essential, I formulated a set of observations which seem to govern Anku's RTP theory. To begin, I examined the first work to which Anku applied his RTP theory to an entire piece, the Adowa example in *Structural Set Analysis of African Music 1*.[7] Some observations were obvious and easy to describe. For example, The RTP grouping of a section is usually preceded by one or more rests:

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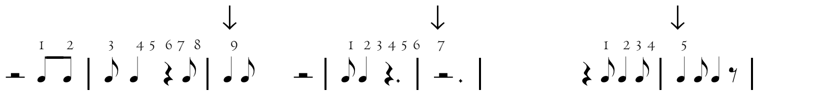
[5] Anku, "Principles of Rhythmic Integration in African Drumming," *Black Music Research Journal* 17.2, pp. 234-35. Arrows indicate the RTP of the timeline.

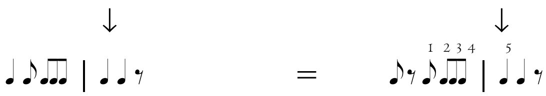
[6] In Anku's transcription, he adds phrase marks over the RTP 2 interpretation of this passage, but he does not say why he hears the passage phrased this way.

[7] The same work also appears in the appendix to his dissertation.



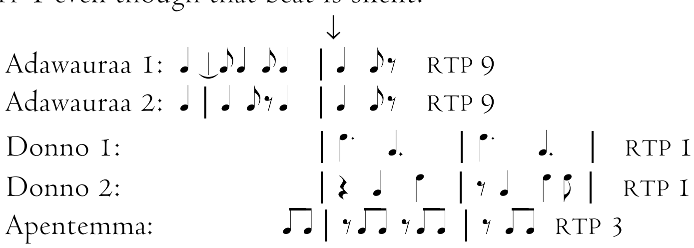
RTP 9, mm. 5–7 RTP 7, mm. 57–59 RTP 5, mm. 170–71

A rest can also be hidden in a transcription by writing it as a quarter note. In the following example, the grouping begins on the first eighth note and the preceding "rest" is hidden in the second beat of the preceding quarter note, which could have also been written as ♪𝄾:



RTP 5, mm. 94–95

A corollary of this observation is that the first beat of a group is nearly always attacked.[8] This is not the case for the 2nd *donno* part of the Adowa support drum complex. Anku considers the line to be in RTP 1 even though that beat is silent:[9]



There seems no reason to put *donno* 2 in RTP 1 except to have it paired with *donno 1*. The line, when heard by itself, does not begin with the strong beat of the timeline. We can conclude that the RTP of a line can be influenced by the RTP of a closely related line or instrument.

Another principle implicit in Anku's theory is that of RTP inertia. If a previous section had a certain RTP, Anku is likely to consider that RTP the RTP of the following section as well. The principle surfaces several times in his Adowa transcription, but is most obviously applied at the end of volume 2, Bawa, where two identical passages, mm. 416-420 and mm. 443-446 are interpreted in two different

[8] The first beat of the RTP line should not be taken as the accent of the line. It might be an upbeat.
[9] Adapted from Anku, *Structural Set Analysis 1: Adowa*, p. 6.

RTP orientations, 7 and 11. The first is RTP 7 because the Anku sees a strong RTP 7 orientation in the preceding two measures, while the four measures immediately prior to m. 443 are in RTP 11.[10]

In order to test the validity of my observations about Anku's RTP presuppositions, I decided to create a group of computer programs which would apply these observations to the Adowa example whose RTP sets he labeled. As the programs were refined and the RTP values generated by the programs approached those found by Anku, I could be reasonably certain that I had either discovered either his unwritten rules or had created some categorization process which produced congruent results. Then I could run the same programs on other pieces to check my work and label their RTPs.[11]

Methodology and Scorefiles

The first step in applying this methodology consisted of encoding the transcribed works into a computer format. Because many scholars believe the choice of encoding influences analytical results, I have explained my encoding ideas in appendix 1. I next encoded rules I inferred about RTP into computer pattern-matching strings. I was somewhat apprehensive at the start about whether Anku would have found validity in this method. However he had a computer program, *powersets.c*, written for him which he included in his dissertation. It may have been that this sort of automatic RTP labeling was an unrealized goal for the process.[12]

I set up my RTP observations not as a group of definite rules which needed to be followed in all cases, but instead as lists of conditions which aid in the classification of data, called "scorefiles." Scorefiles are used in many computer algorithms, such as junk e-mail filters and optical character recognition programs. Each condition may or may not be met by any particular piece of data, but taken

[10] [*] In the Bawa interpretation, Anku emphasizes the derivation of certain RTP orientations from the dance steps accompanying the music. These dance steps are not transcribed.

[11] [*] Analyzing the results of this step is important for avoiding the classic error of evaluating an algorithm based on its ability to regurgitate the analysis used for training. In the absence of a large body of labeled examples not used for training, this step of the paper is less complete than I would have liked.

[12] Anku, "Procedures in African Drumming," pp. 272ff.

as a whole, the conditions can produce a likely interpretation of the data. A scorefile which attempted to find verbs in simple sentences might contain lines looking something like these:

```
word ends in "ing"                 : Score 40
word ends in "ed"                  : Score 20
word is the second in the sentence : Score 5
word is preceded by "the"          : Score -100
```

In this case, a word ending with "ing" is identified as twice as good an identifier of a verb as the ending "ed" and eight times as good as the merely being the second word in a sentence. If a word is preceded by the word "the," it is identified as highly unlikely to be a verb; this is encoded by the score of negative 100. We then would add up the scores and define a cutoff point, say 20 points. Any word scoring at or above 20 would be labeled a likely verb, any below 20 would be unlikely to be a verb.[13]

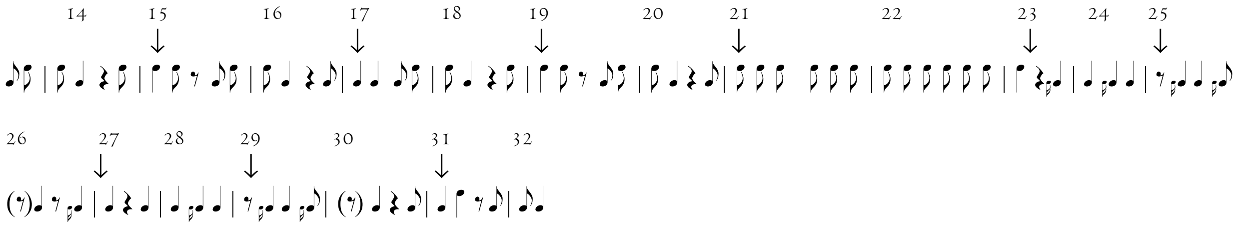
Clearly, these are not all the rules needed to identify even a majority of verbs, and it is likewise obvious that no scorefile could identify every verb in every sentence of the English language without error. However, a relatively short scorefile which took account prepositional phrases, adverbial endings, and dependent clauses could identify a rather high percentage of verbs.

The adowa.score file in appendix 3 attempts to apply the same process to the labeling of RTP sets in Anku's *Structural Set Analysis 1: Adowa* using 26 conditions. The program in appendix 2 then outputs the following pieces of information:

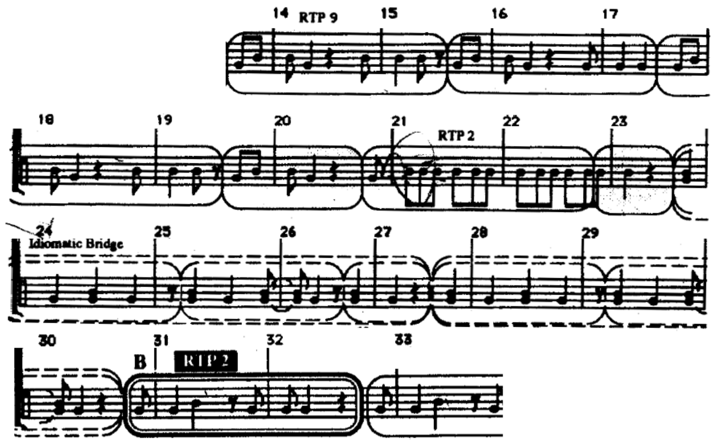
1. The most likely RTP label for that time span
2. A score roughly measuring how certain it is about that label
3. Other RTP labels which might be other interpretations of the passage
4. (for Adowa dances only): whether or not the passage is an "idiomatic bridge," a specific type of transition between sections.

[*] For this passage from Adowa (downward and upward stemmed notes are low and high attacks, respectively. ♩ indicates a flam):

[13] [*] The classification does not need to be binary, nor is it so in the programs used for this paper. There could be many possible results and a level of certainty about the results can be given.



which Anku labeled in this way:[14]



the program produced these results:

beginning set m.	*strongly heard RTPs*						*bridge = i*	*highest score* *highest RTP(s)*
{0005-013}		03			09			{135--,09}
{0006-015}	02				09			{138--,09}
{0007-017}	02				09	11		{155--,09}
{0008-019}	02				09	11		{100--,09}
{0009-021}								{ 31--,02}
{0010-023}								{ 0--,00}
{0011-025}		03					i	{ 45--,03}
{0012-027}								{ 8--,03,12}
{0013-029}							i	{ 15--,02,06}
{0014-031}	02		06					{141--,02}
{0015-033}	02		06	08		11		{186--,02}

The program labeled the shift from RTP 9 to 2 at the same place as Anku but went further to say that RTP 2 was much less strongly felt than the preceding RTP 9. The idiomatic bridge was identified in some places but not consistently. (Of the 26 rules used to label passages, only the algorithm to identify idiomatic bridges was specific to Adowa. These bridges are nine beat groups which are repeated one or more times with intervening sets of varying length. The intervening sets

[14] Anku, *Adowa*, pp. 10–11. I have erased the bell line from this transcription.

allow for transitions between RTP orientations.) A complete list of rules used to label Adowa is found in appendix 3. I present two concepts and their encoding below:

Example concept 1: A 12-beat pattern which is identical to the following 12 beats is roughly twice as strong an indicator of RTP than a 12-beat pattern which has only one difference from the following 12 beats.

Example encoding: (-c1 indicates the line is part of the first rule, -c2 the second, and so on)

```
-c1 word 12          # Consider two sets of length 12
-c1 gap 0            # which are consecutive (no gap between them)
-c1 score 20         # if they are identical, give this RTP a score
                     #      of 20

-c2 word 12          # Consider two consecutive sets of length 12
-c2 gap 0            #
-c2 error_exact 1    # if they differ only by one attack
-c2 score 10         # give this RTP a score of 10
```

Example concept 2: Inertia. When a 6-beat pattern matches a 6-beat pattern 6 beats later, we should consider the current RTP more likely to be the RTP for the next pattern, and to a lesser extent for the pattern after that.

Example encoding:

```
-c1 word 6           # Consider two sets of length 6
-c1 gap 6            # separated by 6 beats.
-c1 score 15         # Score 15 if the sets are identical

-c2 copy 1           # Same as rule 1 (two sets of len. 6, 6 beats apart)
-c2 score 6          # Give 6 additional points
-c2 walk_offset 12   #    to this same RTP one timeline (12 beats) later

-c3 copy 1           # And give 3 additional points
-c3 score 3          #    to the same RTP two timelines (24 beats) later[15]
-c3 walk_offset 24
```

I tried to emphasize common sense and accuracy of results at each stage of encoding the music and writing the programs, seeking to avoid Meki Nzewi's well-founded criticism of "human scientific wizards who abdicate their senses to the wonder machines, and, thereby, miss simple, common sense and natural solutions to natural human problems."[16]

The program is able to label correctly ~60% of Anku's RTP groups. When RTP orientations considered by the program to be second or third choices are added, the percentage jumps to ~75%.

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[15] Other possible modifications were:

```
rfb_attack 1          # require the first beat of this RTP to be attacked
rp_silence 1          # require the beat preceding the first beat to be silent
tone_insensitive 1    # when comparing two sets, ignore differences in pitch
```

[16] Nzewi, *African Music: Theoretical Content and Creative Continuum: The Culture-Exponent's Definitions,* (Olderhausen: Institut für Didaktik popularer Musik, 1997), p. 23.

Some passages were consistently not identified with Anku's labels, such as the passage from mm. 208–214 which seems to rely completely on inertia as a reason for his label of RTP 1. It would be very difficult using simple comparisons to increase the weight of inertia for this section without disrupting the interpretation of the rest of the piece.

Works analyzed with this process

In selecting a group of works to examine next with the aid of the programs, I uncovered a number of somewhat obvious rules governing the applicability of RTP labeling. In order for a repertory to generate a meaningful RTP analysis it had to meet certain basic criteria. First, it needed to have a recurrent beat whose location was agreed upon by the players. Many African drumming types, such as the famed "talking drum" repertories, are in a free rhythm that eludes this sort of recurring set analysis. Secondly, the orientation of a line with respect to the strong beat must change within the work. The support drum parts, for instance, often do not shift with respect to the timeline, and many songs are in a single RTP throughout.[17]

In this paper I looked exclusively at the *atumpan* (the double master drum) part of Adowa. I have not done any of my own transcriptions so I am relying completely on the work of other scholars. Because my work is on "norms of behavior" for this repertory in the aggregate, it is somewhat resistant to small inconsistencies between or within a work. However, in a larger study it would be preferable to create and verify all the transcriptions from source recordings.[18] I relied on the transcription work of three scholars, Anku, through two works, one in his *Structural Set Analysis* and another, unanalyzed, in his dissertation, David Kaminsky, who has kindly lent me an unpublished transcription of Adowa, and

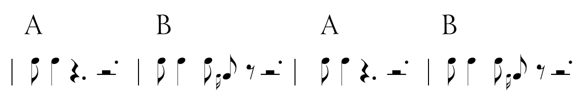
[17] The study of RTP frequency within a repertory of group of repertories within a region might form an interesting study. This paper, like those of Anku, examines RTP use only within an individual piece.

[18] [*] Kofi Agawu has fashioned a rebuttal to the argument that personal recording of performances necessary for analysis in his recent book, *Representing African Music: Postcolonial Notes, Queries, Positions,* (New York: Routledge, 2003).

Kongo Zabana who published a transcription of an *atumpan* part in his *Africa Drum Music: Adowa*, in 1997.

Structural Levels of Sets

Anku showed similarities between pairs of sets arranged like ABAB, where A and B are related but distinct rhythmic sets. He described A and B as being in an antecedent-consequent relationship. The analogy holds in that the beginning of the sets are usually the same but the ending of the sets are distinct. But it fails in that there is not a standard relationship between the endings of the two sets, unlike the dominant to tonic relationship in antecedent-consequent tonal phrase structure. Further, significant deviations between the two sets can occur quite early. An example of this can be seen in Anku's *Structural Set Analysis 1*, mm. 58–65 where we find the differences in the *atumpan* line on the fourth and fifth beats of a 12 beat line:



Anku also looked for similar "pattern synthesis" on the level of the single pattern (where A and B are six beat sets) as well as on the half-pattern level.[19] This division seems to recall his first principles of pattern formation stated in 1988 where he wrote that "all patterns are grouped into four equidistant 'felt beats'."[20] Anku dismisses with little comment divisions of 12 beat patterns into three equal parts ($\frac{3}{4}$) or into unequal divisions.

The most critical omission in Anku's structural levels of sets is the absence of a 2:3 or 1:3 level. At the 2:3 level, two statements of a repeated phrase are heard for every three statements of the

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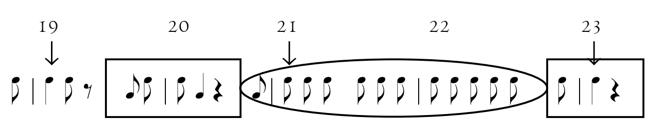
[19] In his 1995 article in *Intercultural Music*, Anku stops here, but in 1997 he writes that there is another level, the "four equidistant division," which is important to consider. Anku, "Cross-Cultural Theory," p. 170. *Idem*., "Principles of Rhythmic Integration," pp. 214–15.

[20] Anku, "Procedures in African Drumming," p. 36. See also pp. 259–260 where he calls the division into four beats "antithesis."

timeline. According to Anku, this is the primary structural level used by the sets in the Bawa dances.[21] When dealing with ratios of set-to-timeline which not relatively prime, such as 3:1, 2:1, 1:1, 1:2, 1:3, etc., the RTP of a passage remains constant so long as a set repeats exactly. However, for relatively prime ratios, such as 4:3, 3:2, 2:3, etc., the RTP of a line rotates by (p/n), where p is the number of beats in the timeline and n is the number of repetitions of the timeline needed to bring the ratio to an integer value. For the 2:3 ratio in Bawa, $p = 16$ and $n = 2$, so RTP rotates by 8 with each repetition of the set $(16 \div 2)$. This can be seen in Anku's chart on pp. 32-36 where RTP 1 is followed by RTP 9 and then back to 1; RTP 5 rotates to 13 and back; and RTP 15 oscillates with 7.[22] That RTP moves even when the drum line is behaving "circularly," that is, repetitively, leads to the conclusion that RTP cannot be interpreted as the a constant value for understanding a long passage of music if relatively prime set-to-timeline ratios are allowed.

Further Comments on RTP

Anku's system of interpolated sets posed a particular problem for reproduction both in the computer encoding and for identification by hand. These are passages where a phrase with a certain RTP orientation is interrupted mid-set by one or more phrases with a different RTP orientation. The remainder of the broken set is then found later in the work, before the next bridge section. An example of this is found in *Adowa*, mm. 19–23:

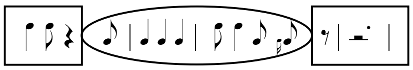


The squares show a set in RTP 9 interrupted by RTP 2. The set containing RTP 9 can easily be heard as being interrupted, but it is difficult to hear the second square as completing the RTP 9

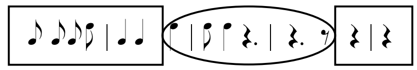
[21] Anku, *Structural Set Analysis 2: Bawa*, p. 11. For Anku, the timeline of Bawa is four quarter notes or 16 sixteenth notes in length, while the prevailing length of the master drum patterns are six quarter notes or 24 sixteenth notes long. If we hear Bawa in a 1:3 ratio, with a timeline that is half as long (2 quarter notes), we have RTP values from 1-8, and the problems of RTP rotation discussed in this paragraph disappear.

[22] The interpretation of the chart for mm. 17^5-20^4 is an error, as will be mentioned below.

pattern, which had previously ended with ♪ | ♪ ♪. In a majority of instances the completion is even more difficult to hear, since the interruption comes after the final attack of the preceding set and thus the complement consists entirely of rests:



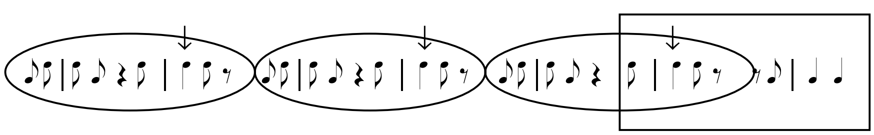
mm. 66–69



mm. 152–56.

Although I do not see the need for most nested interpolations, even if one accepts them, Anku's analysis of Adowa's nested interpolations from m. 36 beat 6 to m. 48 beat 5 seems needlessly complicated. Instead of nested incomplete RTP 4 and RTP 11 sets, an analysis that used one complete and one incomplete RTP 8 interpolation and two complete RTP 10 interpolations would also summarize the rhythms being heard with one fewer "nesting" relationship.

Anku allows for the use of "pivot sets" which function both in a prior and upcoming RTP orientation in an analogous way to how pivot chords function to move from one key to another.[23] I have found pivot sets to be more useful than RTP interpolation for explaining alternation from one RTP to another and feel this aspect of Anku's work should be expanded. Unlike pivot chords, the pivot set is usually identical to the main material of the preceding section:



The circles indicate an RTP 9 interpretation while the box indicates RTP 2. Five eighth notes are simultaneously in both RTP 9 and 2.[24] It is possible, and to me quite audible, that RTP 2 is heard quite a bit earlier in the passage and rather than having a sudden pivot to RTP 2 the passage slowly slips into that orientation.[25] The interpretations presented by my programs reflect the process of slippage

[23] Anku, "Cross-Cultural Theory of Rhythm," pp. 178-80.

[24] Note that a pivot set is thus of different length than the other sets in the piece.

[25] David Locke's discussion of the "aural illusions" of polymeter and temporary cross-relations, while not discussing precisely this phenomenon, should be referenced here if only as a caveat. Locke states that

by labeling long passages as audible in both the preceding and following RTP.[26]

The charts labeled "analysis" in Anku's works are more properly summaries of annotations he has made to his transcriptions. In the pages following the digital transcriptions and reductions provided in *Structural Set Analysis of African Music,* volumes 1 and 2, he does not generalize the results into basic principles of African rhythmic conception or formal composition. For example, his work shows clearly that in neither "Adowa Verbal Themes" (vol. 1, pp. 20–21) nor the interpolated patterns are RTP 3, 6, or 10 used. The relationship between RTP 2 and 8 is never explored despite its seeming importance in Adowa.[27] Anku further mentions in summary 3 of volume 1 that "whenever it [the idiomatic bridge] is played, a new set orientation is naturally expected," but he does not explain why theme D (RTP 5) can be followed by theme E (also RTP 5).[28] The larger question of whether pieces with many different RTP orientations are more complex or more interesting than those with few or one could have been taken up in the analysis as well.[29]

Anku's summary also makes clear that RTP does not seem to show the types of cognitive isomorphisms with respect to pitch and tonal organization that other scholars have found in timeline patterns. I was encouraged to look into this by the opening of Anku's Adowa analysis which shows

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although Africans do occasionally perceive a shift in beat orientation or even meter, this observation is made much more commonly by Westerners. In this case, my ears may be deceiving me. (Locke, *Drum Gahu: A Systematic Method for an African Percussion Piece* (Crown Point, Indiana: White Cliffs Media Company, 1987), pp. 16-31).

[26] [*] I worked out the idea of slippage between keys via extended pivot regions in an unpublished study of automatic labeling of key areas in Bach chorales completed in 2003 with Johnny Carlsson, DU Jinyan, Vitaly Feldman, and Chang Liu.

[27] In Bawa dance forms, a 3:2 ratio of the lengths of master drum sets to the time line produces alternating sets which are RTP compliments mod 16; this could be an expression of the same phenomenon mod 12.

[28] Anku, *Structural Set Analysis 1*, p. 25. [*] Since Anku's transcriptions are in some ways idealized transcriptions of possibly imperfect performances, we cannot rule out the possibility that an error in performance has occurred. Alternatively, the use of the idiomatic bridge in this case could be intentionally in conflict with the expectations of the listener, in the same way a deceptive cadence works in tonal music.

[29] There are several errors in the summary which impede its usefulness. Mm. 71^6-73^5 should be an RTP 8 interpolation while mm. 79^5-83^4 should be an RTP 12 interpolation. Both sets are correctly marked in the score. In the Bawa volume, mm. 17^5-20^4 are marked as RTP 1 when they are clearly RTP 5's mod 16 compliment, RTP 13. This is likely a typographical error.

major areas of RTP 9, 2, and 7 in succession from section **A** to **B** to **C**. This progression corresponds to a shift of seven beat classes to the right in each stage—or as an isomorphism to pitch, a chain of ascending perfect fifth relationships. However, I have not seen this connection between RTP of successive or strongly emphasized patterns in other pieces.

If the relationship of RTP to one another does not show pitch isomorphisms, the frequency of certain patterns in certain RTPs is even less encouraging: showing a negative relationship. That is to say RTPs corresponding to common tonal consonances are uncommon and those corresponding to dissonances are common. RTP 11 (corresponding to minor 7) is very common, as is RTP 7 (tritone), while RTP 1 (unison or tonic) is quite uncommon in Adowa, though it is more common in the song patterns. This observation also affects the work of other scholars. The lack of large-scale supporting evidence for connections between rhythmic sets and pitch structures does nothing to take away from the work of Jeff Pressing on the subject, since Pressing looked at isomorphisms within sets and scales.[30] Jay Rahn's work, by contrast, depends on cyclic structures across repeated sets to generate similar pitch and rhythm structures.[31] Although my findings do not directly attack the validity of his work, it should cause us to reevaluate whether large collections of rhythmic sets, such as occur in a whole piece, have similarities to pitch structures.

I have a concern with the term RTP itself. "Regulative time point" seems to suggest that the point which is the performer's reference for a given phrase is always the following strong beat. The possibility of a regulating preceding beat is not discussed.[32] I feel it is at least equally likely to hear the following passage RTP 11 passage as "RTP negative 1," that is, beginning on the first beat after the preceding downbeat:

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[30] Jeff Pressing, "Cognitive Isomorphisms between Pitch and Rhythm in World Musics: West Africa, the Balkans and Western Tonality," *Studies in Music* 17 (1983), pp. 38-61.

[31] Jay Rahn, "Asymmetrical Ostinatos in Sub-Saharan Music: Time, Pitch and Cycles Reconsidered," *In Theory Only* 9.7 (March 1987), pp. 23-36.

[32] The ideas in this paragraph were worked out through discussions with David Kaminsky.

The problem arises both from the word "regulative" and that the same term, RTP, is used both for the downbeats themselves and for the lines' and sets' relationships to those beats. I also feel that the term "structural set analysis" draws a connection with pitch set analysis which does not hold in a fundamental way. Traditional atonal set-based analysis is concerned not only with the properties of one particular set, but with the relationships between seemingly different sets within a composition; by taking up this term for his own work Anku is promising the reader such an investigation, which does not come to light in his publications.

Prime forms[33]

My wish to discuss Anku's work on prime forms stems partly from the amount of interest it has spurred on the mto-talk Internet mailing list.[34] Anku introduces a new terminology into the discussion of interval classes and prime forms beyond what Pressing and Rahn have used.[35] He indicates with angle brackets "cross-sets" which incorporate tuplets into the basic rhythmic structure. For example, he would encode the following pattern as [1111 1111 <111> 1111]:

Likewise for a twelve-beat set, [111 <22> 111 111]:

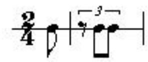
---

[33] Anku's prime forms are more similar to Forte's normal vectors than to his prime forms. Anku acknowledges this in his MTO article (¶ 14). To convert a Forte normal form into a prime form, inversions of the set must be considered equivalent. Anku, wisely, does not do this for his sets. One difference between Anku's prime forms and Forte's normal vectors should be noted. Anku removes the requirement that the distance between the first and last elements of the set be minimized (c.f., Forte 1974, pp. 3-4, Anku 2000, ¶ 14).

[34] It must be noted though that, with the exception of Jay Rahn and Richard Cohn, most of the contributors were not aware of isomorphisms between Western scale patterns and African rhythmic patterns before reading Anku's paper. Anku deserves credit more for bringing this idea to a wider public than for the originality of the isomorphisms itself.

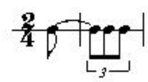
[35] Pressing, "Cognitive Isomorphisms," op. cit. Rahn, "Asymmetrical Ostinatos," op. cit.

Anku indicates tuplet rests by writing a figure as if it were attacked but placing it in parentheses (figure 14 in Anku 2000):



2<(1)11>

If a note is sustained into a cross-set, this is indicated by a hyphen placed between the preceding figure and the opening angle bracket (figures 13 in Anku 2000):



2-<(1)11>

There is an important inconsistency which arises with Anku's modifications to beat class description. I have no quibble with his orthography *per se*; the use of angle brackets seems to suggest coloration and thus seems natural to anyone having worked on rhythmic patterns in late-Medieval Western music. My criticism is with the introduction of note length into a system which was previously concerned only with intervals between attacks. Consider, for example, the beat class set (bc set) whose prime form is [1221222].[36] We have no way of knowing whether the musical line represented by this bc set is ♪♩♩♪♩♩♩ or ♪♪𝄾♪𝄾♪♪𝄾♪𝄾♪𝄾 or 𝅘𝅥𝅯𝄿 𝅘𝅥𝅯𝄾. 𝅘𝅥𝅯𝄾. 𝅘𝅥𝅯𝄿 𝅘𝅥𝅯𝄾. 𝅘𝅥𝅯𝄾. 𝅘𝅥𝅯𝄾. or any combination of the three. Anku's notation creates a distinction between held notes and rests but only in the case of entering or leaving a cross-set. His hyphen conflicts with the purpose of bc sets and should be dropped.

Additionally, with Anku's angle bracket notation, we can now no longer perceive the distance between attacks by looking at a single digit alone. To gauge the distance between the first and second attacks in his bc set "2<(1)11>" we need to parse "2<(1)". Further, the precise duration of a figure within an angle bracket can only be determined by knowing the prevailing beat orientation of the work. "2" by itself always means two beats whether we are in a 12 or a 16 beat system. But "2" within angle brackets means eight-thirds of a beat in a 16 beat set but only three-halves of a beat in a 12 beat set.

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[36] This beat-class set is isomorphic to the pitch-class set of the diatonic scale. A confusing typographical error appears in the *Music Theory Online* article, figure 11b, where this bc set [1221222] is labeled [121222].

For the purposes of bc set manipulation and comparison, fractions, though initially more difficult to read, present a better alternative to Anku's angle brackets:

[1111 1111 <111> 1111] → [1111 1111 ${}^{4}/_{3}\,{}^{4}/_{3}\,{}^{4}/_{3}$ 1111]
bc 16: [2-<(1)11>] → [${}^{10}/_{3}\,{}^{4}/_{3}\,{}^{4}/_{3}$]
bc 12: [2-<(1)1>] → [${}^{7}/_{2}\,{}^{3}/_{2}$]

With the exception of these short examples, the new bc set notations he devises are never applied to music; Anku seems to add them for the sake of completeness rather than for any illumination it gives to his published analytical work.

In his dissertation and in two articles published since (1995, 2000), Anku has labeled the prime forms of various sets discussed in his works. He has not however, labeled any work with more than nine sets, nor has he given any indication of the usefulness of labeling as an analytical tool. I have labeled all the prime forms in the transcriptions of Adowa *atumpan* parts provided by Anku (1988, 1992), Kongo Zabana (1997), and David Kaminsky (2000).

A small, but not inconsequential, number of sets with the same prime form but different rotations appear in the same piece.[37] Some bc sets such as [22224] are not surprising to find in several rotations. This is a common form of the idiomatic bridge, which we have already seen is actually a nine beat set. Thus its migrating position is what allows it to function as a bridge between RTPs, especially those 3, 6, or 9 beats apart. Multiple rotations of other prime forms are more interesting. For example, [15222] appears in two forms <22215> and <21522> in Anku's 1992 transcription:

<22215>: mm 127-28, 131-32: ♩♩♩♪♩𝄽.
<21522>: mm. 143-44, 155-56, 175-6: ♩♪♩𝄾𝄽♮♩♮♩

There are three variants of the [11145] bc set found in the Adowa dance. In all cases, the pattern is analyzed as the compliment of some preceding (often distant) incomplete set. In the first

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[37] The program used to generate these results is found in Appendix 5 and the complete results for Anku 1992 appears in Appendix 6.

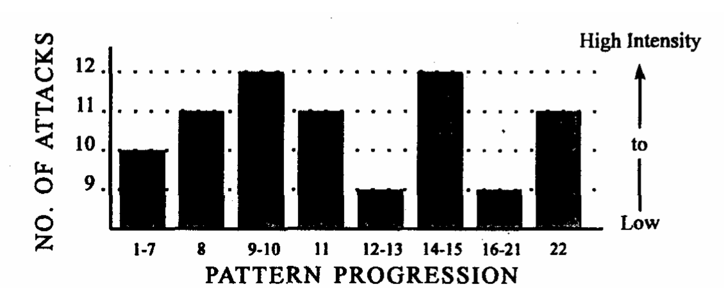
case, it participates in the triple nested interpolation I discussed earlier. For these sets alone, I feel the prime form linkage between them is stronger than the connection to the immediate RTP group they are said to compliment:

mm. 47-48 <51141>: 
mm. 83-84 <11415>: 
mm. 149-50, 159-60 <15114>[38]: 

The analysis of rotational equivalency within the African repertory is only beginning. Development of these analytical tools will require looking at how prime-form equivalent sets interact between instruments and across genres. It will also need to be subjected to the difficult conceptual test (as some have attempted in pitch class theory) of whether it is indeed perceivable by listeners and performers. Here, Anku's reticence to allow for inversional equivalence will probably be rewarded.

Attack Density as a Measurement of Intensity

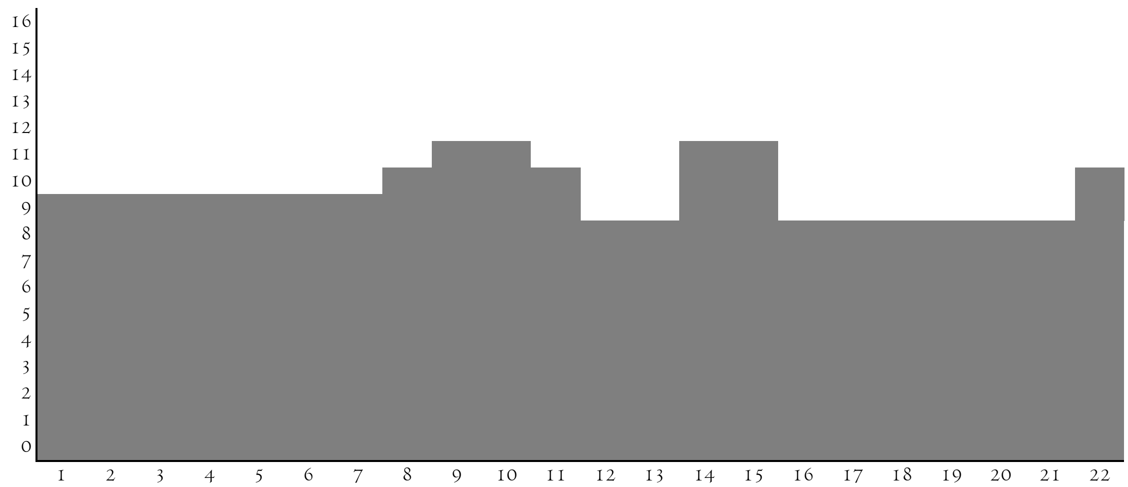
In his 1995 article in *Intercultural Music*, Anku counts number of attacks in each set as a measure of the intensity throughout a region. His analysis of an unidentified 16-beat master drum pattern is presented below:[39]



[38] This appears in the music with the two beats of silence at the front of the set. There is no way to encode this in Anku's system

[39] Anku, "Towards a Cross-cultural Theory of Rhythm in African Drumming," in *Intercultural Music*, volume 1, edited by Cynthia Tse Kimberlin and Akin Euba (Bayreuth, Germany: E. Breitinger, 1995), p. 195. The same chart appears in Anku, "Procedures in African Drumming" (1988), p. 287.

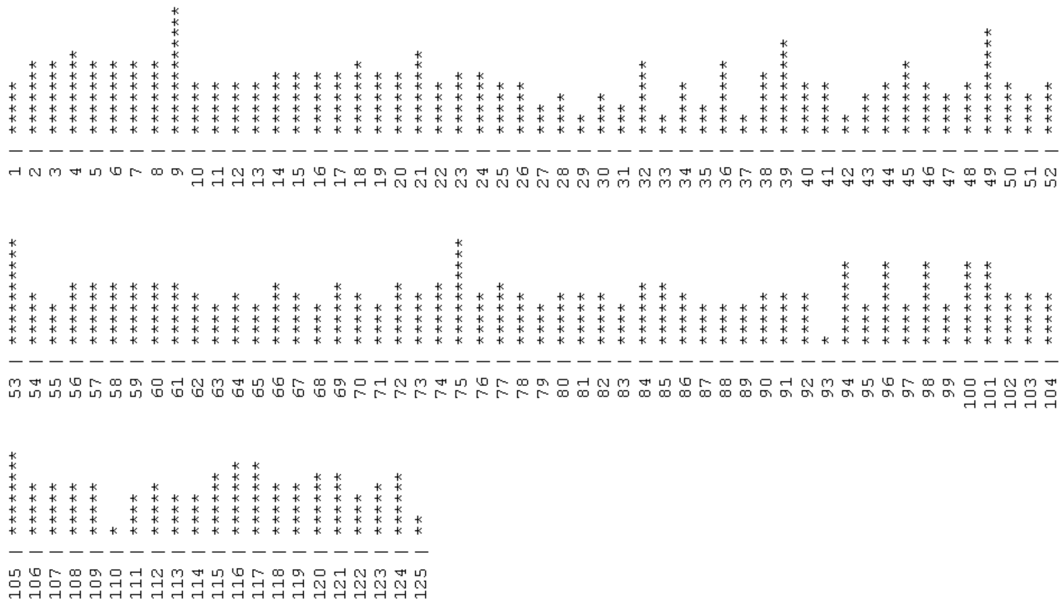
What might look like a coherent pattern of increasing intensity followed by decreasing intensity followed by alternation between dense (intense) and sparse patterns is much more likely statistically insignificant fluctuations. Anku's intensity scale ranges from nine to 12 attacks while the theoretical range for this pattern is zero to 16. He also joins successive sets which have the same number of attacks, which tends to overemphasize change. When plotted on a scale that shows the full range of possible numbers of attacks and gives each pattern equal weight, the insignificance of the fluctuation becomes clear:



It would seem that, at least in this piece, the number of attacks in each set is not a good indication of the intensity of a section. In fact, the stability of the number of attacks might be a feature of this performance of Bawa.

In order to see if there were larger patterns, I constructed another program to give the number of attacks in each set of any of the musical examples encoded as above.[40] The graph for Anku's Adowa example in *Structural Set Analysis 1* is presented below:

[40] This program is found in Appendix 5. The output of the program must be rotated 90 degrees to get the output below.



In examining a larger composition little more is gained over Anku's analysis of a shorter work. No large-scale patterns of attack density were to be found in the transcriptions of *atumpan* parts by Anku (1988, 1992), Zabana (1997), or Kaminsky (2000). A few small-scale patterns of interest can be noted: The "T⊃me kume menie" passage maintains the highest sustained level of rhythmic density in Adowa: seven attacks per 12-beat set. When we view the density map with adjacent, identical sets reduced to one point, as Anku reduced them in his chart, another phenomenon emerges. The idiomatic bridges are the only places where the rhythmic content of the set changes but the number of attacks in the sets remains constant. This observation would have to hold true for a larger repertory of performances before it could be strongly asserted. Anku's has continued to assert the importance of attack density despite a lack of further supporting evidence.[41] His chart, and my work expanding it, reveals no coherent large-scale patterning in the music, except perhaps a *flatness* of attack density in

[41] Although his original use of attack density preceded by four years Richard Cohn's use in his 1992 article on attack density and beat-class transposition in the early minimal music of Steve Reich, Anku might have been encouraged to continue using the tool because of the powerful results Cohn was able to achieve with it. Cohn was able to show distinct patterns of attack intensity which were consistent with patterns of pitch elements and which were similar between the two works studied, *Phase Patterns* and *Violin Phase*. See Cohn, "Transpositional Combination of Beat-Class Sets in Steve Reich's Phase-shifting Music,"*Perspectives of New Music* 30.2 (Summer 1992), pp. 146-177, and especially, pp. 154-158.

certain works, and thus should be shelved until a useful explanation of the density patterns can be found.

Non Set-Oriented and Borrowed Theories

Not all of Anku's published ideas concern the organization of African music into sets, nor are they all as original and pervasive in his writings as his RTP theory, but they compliment and give a wider philosophical and sociological background to his other theories.

The idea that African music is circular generated the title of Anku's 2000 publication, "Circles and Time: A Theory of Structural Organization of Rhythm in African Music."[42] He notes that the concept is not original to him but has a long history within the field. The originator of the term "timeline" was, in fact, Anku's advisor at University of Pittsburgh, J. H. Kwabena Nketia. Anku's principal contributions to the concept of circularity in African music are the diagrams he uses to describe the relationships between patterns. These diagrams closely resemble those Meki Nzewi published three years earlier.[43]

By circular, Anku means that a greater degree of immediate repetition is demanded by the style than is necessary in Western music. This is especially true for the instrument or instruments playing the timeline and those instruments playing some inner patterns. Although he allows for different starting points for the same timeline in the music of different cultures, he does not use the term "circular" to mean that music or a line of music can begin in any arbitrary location. Nor does he mean that the notes of a line are not hierarchically organized—circular equality *à la* the Knights of the Round Table. In fact, Anku's RTP theory, examined above, depends on neither of these aspects of

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[42] *Music Theory Online* 6.1 (January 2000) http://boethius.music.ucsb.edu/mto/issues/mto.00.6.1/mto.00.6.1.anku.html.

[43] Nzewi, *African Music: Theoretical Content and Creative Continuum: The Culture-Exponent's Definitions* (Olderhausen: Institut fur Didaktik popularer Musik, 1997). [*] David Rycroft's also used a form of circular notation, which was cited in the article "Africa" in *The New Grove Dictionary of Music and Musicians* (London: Macmillian, 1980). I agree with Kofi Agawu's unpublished observation that circularity in African music is often asserted but neither systematically studied nor well-defined.

circularity being true. The first attack's distance from a point of highest importance is the most prominent of Anku's theoretical constructs.

Another theory which appears intermittently in Anku's writing is the rhythmic basis of drumming in speech and song. This has been explored more deeply by other writers, most notably Kofi Agawu.[44] To neglect textual influences such as the "T⊃me kume menie" opening in Adowa would be to deny an obvious controlling element in that musical form. But other repertories which interest Anku, such as the Bawa dances, have rhythms which he says are "not known to be text-bound."[45] Thus, a vocal background for African music cannot fit into his universal set theories and seems almost a digression when he mentions it in that context.[46]

This paper also barely skims the ethnographic and historical research of Willie Anku. For the first, he has conducted work on drumming communities in Pittsburgh which he published as part of his 1988 dissertation.[47] His historical ideas were asserted in the conclusion to his 1995 article in *Intercultural Music* where he suggests that population migration and cultural and linguistic sharing during a past "period of diffusion" created a common African culture.[48] This shared culture is, of course, necessary for any generally applicable theory of African rhythm.

The theories of Willie Anku offer fresh attempts at providing analytical frameworks for rhythm across repertories in sub-Saharan Africa. None of the problems I feel exist Anku's work are insurmountable, and my concerns with most of those could be quickly rectified by applying the theories to a

[44] Agawu, *African Music: A Northern Ewe Perspective* (Cambridge: Cambridge University Press, 1995), pp. 27-60.
[45] Anku, *Structural Set Analysis of African Music 2: Bawa* (Legon, Ghana: Soundstage Productions, 1993), p. 37.
[46] [*] It would be most helpful to know whether Anku considers the textual cues to generate the music or whether they are mnemonics applied to rhythms conceived purely musically.
[47] Anku, "Procedures in African Drumming: A Study of Akan/Ewe Traditions and African Drumming in Pittsburgh," (Thesis (Ph.D.), University of Pittsburgh, 1988), part II.
[48] Anku, "Towards a Cross-Cultural Theory of Rhythm in African Drumming," in *Intercultural Music*, volume 1, edited by Cynthia Tse Kimberlin and Akin Euba (Bayreuth, Germany: E. Breitinger, 1995). p. 196-97.

wider range of works and generalizing the results. RTP is at the very least a useful tool for understanding rhythmic changes within pieces in the Adowa repertory. A modified version taking into account sets to bell pattern ratios of 3:2 would improve RTPs utility for discussing the Bawa repertory. The current problem of oscillation between mod 16 complimentary sets in Bawa undermines attempts to consider sections of a work as dominated by certain RTP. There is also a continuing role for ethnic perception and ethnographic interview for RTP theoretical work. Are there places in pieces where a player with a changing RTP, such as the master drummer, feels that his part is grooving with, or feels tension with, one or another of the support drum lines? Or vice-versa for a support drum player? If so, is there a pattern to the RTP relationships between the two parts which coincides with these feelings?

Anku's second major principle, prime forms, could be more closely tied to RTP. A search for whether certain prime forms, or interval vectors, occur more commonly in certain RTP designations could explain why some verbal themes always have particular RTP orientations. It might also show more generally which sets have prime forms that occur only in one rotation and which bc sets, such as [1221222], occur in many rotations both within a piece and throughout Africa?

It is my hope that answering these questions via an investigation of a wide repertory of instrumental and vocal works will enable scholars to go beyond labeling and classification of repeated sets and move to reproducible and analytically fruitful examinations of African rhythm.

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APPENDICIES

Appendix 1: Representations of RTP

The RTP theory can be applied to rhythmic patterns regardless of the notational system used to notate them. The first two figures below show a single melodic line expressed in four different RTP using two different notational systems. The first system is standard Western notation though with barlines removed:

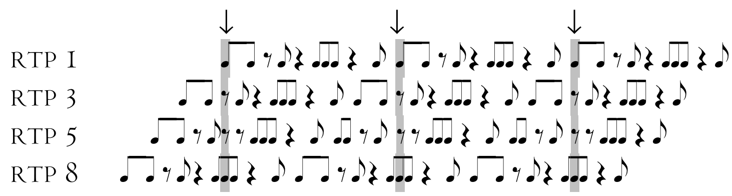

Figure 1a: A demonstration of Anku's RTP sets using traditional notational systems but without bar-lines.

The second is James Koetting and Philip Harland's Time Unit Box System (TUBS).[49]

	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6
RTP 1	•	•		•			•	•	•			×	•	•		•			•	•	•			×	•	•		•		
RTP 3		•			•	•	•			×	•	•		•			•	•	•			×	•	•		•			•	•
RTP 5			•	•	•			×	•	•		•			•	•	•			×	•	•		•			•	•	•	
RTP 8	•	•			×	•	•		•			•	•	•			×	•	•		•			•	•	•			×	•

Figure 1b: A demonstration of Anku's RTP sets using a TUBS-based notational system using a the same rhythmic pattern. × indicates a stroke not always present in the quoted pattern. Shading is used to delineate repetitions but is not an essential part of the encoding.

Although the TUBS system can be more difficult to read at first, particularly where long rests or strings of consecutive attacks occur, it has some important advantages for evaluating Anku's work. The first is that it demonstrates well how a change in perception of the position of the fundamental beat—which Anku asserts is "circular," that is, ethnically constructed—necessitates only a change in the numbering of the beats in the top row of the chart and in the RTP numbers in the left column. Unlike Western notational systems, no change in beaming or barring would be necessary:

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[49] Set forth first in James Koetting, "Analysis and Notation of West African Drum Ensemble Music," *Selected Reports in Ethnomusicology* 1.3. pp. 115-46.

	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
RTP 4	•	•		•			•	•	•			×	•	•		•			•	•	•			×	•	•		•		
RTP 6		•			•	•	•			×	•	•		•			•	•	•			×	•	•		•			•	•
RTP 8			•	•	•			×	•	•		•			•	•	•			×	•	•		•			•	•	•	
RTP 11	•	•			×	•	•		•			•	•	•			×	•	•		•			•	•	•			×	•

Figure 2: The same collection of repeated patterns as figure 1 presented in an ethnocentric framework where the regulative time point is conceived three beats later, such as might happen when two listeners with different musical upbringings listen to the same performance. Note that the RTP values of the four patterns are increased by 3 (mod 12) from the previous figure, but the shading remains the same.

Further, the TUBS system makes equivalent patterns such ♩♩ and ♪𝄾♪𝄾. There is thus less variation between different scholars' transcriptions of the same or similar pieces. Some sections which are the same or nearly the same between pieces, such as Adowa's idiomatic bridges, particularly benefit from the notational standardization TUBS brings. Another important advantage of the TUBS system is that it can be easily encoded in a computer-readable format, such as the one presented below:

RTP 1: 110100111002110100111002
RTP 3: 010011100211010011100211
RTP 5: 001110021101001110021101
RTP 8: 110021101001110021101001

Figure 3: One way the patterns in figure 1 might be encoded into a computer-readable system, similar to that used by rtp.pl (see appendix 2). This system resembles TUBS in that each base-beat occupies an equal unit of space; in this case, one digit. Normal stokes in this system have been encoded as "1" while optional strokes have been entered as "2". Base-beats which are not articulated are represented by "0". The principal disadvantages of this notational system, legibility and ease of pattern recognition, are not problems for a computer. Computer pattern-matching is actually substantially enhanced by this encoding.

Appendix 2: Automatic labeling of RTP via rtp.pl

I have included the listings of the programs I used for this paper so that other scholars with little or no programming experience could examine the validity of these results on their own transcriptions. Other programs following the same principles should give similar results.

This is the version of rtp.pl as of the 18 May 2000. Some examples may have been generated with different versions and thus give slightly different results. The same holds true for other files and output in the appendices.

```
#!/usr/local/bin/perl

use strict;
use POSIX;
```

```
##############################################################################
####                                                                         #
####  rtp.pl --   A program to find RTP patterns in African music based on   #
####             scorefiles                                                  #
####                                                                         #
####       usage: perl rtp.pl -fSCOREFILE                                    #
####                                                                         #
##############################################################################

## Initialize

my $bc = 12;
my ($word_length, $gap) = (12,0);
my ($measure_size, $initial_offset) = (6,5);
my $print_summary = 0;
my $print_all_words = 0;
my $print_score = 0;
my $minimum_score = 1;
my $require_firstbeat_attack = 0;
my $require_preceeding_silence = 0;
my $title = '';

my @comp_array;
my $line = '';

#### Process Arguments

foreach (@ARGV) {
  if (/-f([\w|\.]+)/) {
    open(SCOREFILE, $1) || die "Couldn't open $1 for reading";
    my @score_contents = <SCOREFILE>;
    close (SCOREFILE);
    foreach (@score_contents) {
      next if /^\#/;
      process_arg($_);
    }
  } else {
    process_arg($_)
  }
}

print_summary() if $print_summary;

#### Run Program

$line ||= join '', <STDIN>;

my $fill_ref = {};
for (my $rtp = 1; $rtp <= $bc; $rtp++) {
  my $striped_line = strip_line($line, $rtp);
  foreach my $comp_ref (@comp_array) {
    compare($striped_line, $fill_ref, $rtp, $comp_ref);
  }
}
print_match($fill_ref, (strip_line($line,0)), $bc);
print ''; ## for the debugger

###############################################################################
####                                                                          #
####  subroutines: process_arg -- sets vars based on cmdline or scorefiles    #
####              print_summary -- sends out what a well commented scorefile #
####                               should already tell you                   #
####                                                                          #
###############################################################################

sub process_arg {
  $_ = shift;
  if    (/^-m(\d+)/)     {$measure_size = $1}
  elsif (/^-b(\d+)/)     {$bc = $1}
  elsif (/^-o([-|\d]+)/) {$initial_offset = $1}
  elsif (/^-w(\d+)/)     {$word_length = $1}
  elsif (/^-g([-|\d]+)/) {$gap = $1}
  elsif (/^-a/)      {$print_all_words = 1}
  elsif (/^-s/)      {$print_summary = 1}
  elsif (/^-ms(\d+)/) {$minimum_score = $1}
  elsif (/^-ps/)     {$print_score = 1}
```

```
  elsif (/^-rs/)     {$require_preceeding_silence = 1}
  elsif (/^-ra/)     {$require_firstbeat_attack = 1}
  elsif (/^-t\s*[\"|\'](.+)[\"|\']/) {$title = $1}
  elsif (/^-rf\s*\"(.+)\"/) {
    open (READIN, $1) || die "Couldn't open $1 for processing\n";
    $line = join '', (<READIN>);
    close (READIN);
  }
  elsif (/^-c(\d+)\s+(\w+)\s+([-|\w]+)/) {
    if (ref($comp_array[$1]) ne "HASH") {
      $comp_array[$1] = {};
    }
    if ($2 eq "copy") {
      my %temp_hash = %{$comp_array[$3]};
      $comp_array[$1] = \%temp_hash;
    } else {$comp_array[$1]->{$2} = $3}
  }
  elsif (/^-c(\d+),(\d+),(\d+)/) {  ## for command line usage
    my %comp_hash;
    $comp_hash{'word'} = $1;
    $comp_hash{'gap'}  = $2;
    $comp_hash{'score'}= $3;
    push @comp_array, \%comp_hash;
  }
}

sub print_summary {
  my $summary_line = "";
  print "\n";
  print "** $title **\n\n" if $title;
  foreach my $comp_ref (@comp_array) {
    printf("Notes to Match: %2d, Gap between matches: %2d, Score: %2d\n",
      $comp_ref->{'word'}, $comp_ref->{'gap'}, $comp_ref->{'score'});
    printf("  Score affects RTP %2d beats later.\n", $comp_ref->{'walk_offset'}) if $comp_ref-
>{'walk_offset'};
    printf("  First beat need not be attacked.\n") if !$comp_ref->{'rfb_attack'};
    printf("  Preceeding $comp_ref->{'rp_silence'} beat(s) must be silent.\n")  if $comp_ref-
>{'rp_silence'};
    printf("  Pattern will be compared without respect to pitch.\n") if $comp_ref-
>{'tone_insensitive'};
  }
  print "\n";
  printf("All phrases will be printed, even ones with no matches.\n") if $print_all_words;
  printf("Measure Size  : %2d, First Measure: %2d\n", $measure_size, $initial_offset);
  printf("Minimum Score : %2d\n\n",$minimum_score);
}

###############################################################################
####                                                                          #
####  strip_line -- eliminates beats from the beginning so that the piece is  #
####              in the proper RTP                                           #
####                                                                          #
###############################################################################

sub strip_line {
  my $line = shift;
  my $rtp  = shift;
  my $trim_amount = 0;
  $trim_amount = ($bc + 1 - $rtp) if $rtp;

  $line =~ s/^.{$trim_amount}//;
  return $line;
}


###############################################################################
####                                                                          #
####  make_binary(line_of_012012_etc)                                         #
####              -- eliminates pitch distinctions if any, returns 3/8        #
####                 groups as binary representations                         #
####                                                                          #
###############################################################################

sub make_binary {
  my $line = shift;
  $_  = $line;
```

```
  if ($bc % 3 == 0) {
    s/(\d{3})/$1 /g;
    s/0/o/g;
    s/\d/x/g;
    s/xxx /7/g;
    s/xxo /6/g;
    s/xox /5/g;
    s/xoo /4/g;
    s/oxx /3/g;
    s/oxo /2/g;
    s/oox /1/g;
    s/ooo /0/g;
#   s/(\d{4})/$1 /g;
    s/(x*o*x*o*x*o*)$//g;
  } else {
    die "I don't know how to deal with bc of $bc\n";
  }
  return $_;
}

################################################################################
####                                                                           #
####  compare -- returns a hashref of words which are identical to the        #
####             following one                                                 #
####                                                                           #
################################################################################

sub compare {
  my $line        = shift;
  my $return_hash = shift;
  my $rtp         = shift;
  my $comp_ref    = shift;

  my $word_size    = $comp_ref->{'word'};
  my $gap          = $comp_ref->{'gap'}   || 0;
  my $score        = $comp_ref->{'score'} || 0;
  my $rfb_attack   = $comp_ref->{'rfb_attack'} || $require_firstbeat_attack;
  my $rp_silence   = $comp_ref->{'rp_silence'} || $require_preceeding_silence;
  my $walk_offset  = $comp_ref->{'walk_offset'} || 0;
  my $begin_offset = $comp_ref->{'begin_offset'} || 0;
  my $step_size    = $comp_ref->{'step_size'} || $word_size + $gap;
  my $tone_ins     = $comp_ref->{'tone_insensitive'} || 0;

     ## suffix -- tag to print for extraordinary circumstances, such as
     ##        -- idiomatic bridges

  my $suffix       = $comp_ref->{'suffix'} || 0;
#  my $ignore_word  = $comp_ref->{'ignore_word'} || 0;

  if ($tone_ins) {
     $line =~ s/0/o/g;
     $line =~ s/\d/1/g;
     $line =~ s/o/0/g;
  }

  for (my $walk = $begin_offset; $walk <= (length($line) - 2*$word_size);
       $walk += $step_size) {
    next if (substr($line, $walk, $word_size) ne
        substr($line,($walk + $word_size + $gap), $word_size));
    next if ($rfb_attack && (substr($line, $walk, 1) == 0));  ## make sure first beat is not
silent
    next if ($rp_silence && (substr($line, $walk-$rp_silence, $rp_silence) > 0)); ## make sure
previous $rp_silence beats are silent
    my $affected_walk = $walk + $walk_offset;  ## a match can give a score to another position
    $return_hash->{$affected_walk}->[$rtp] += $score;
    $return_hash->{$affected_walk}->[$bc+1] = $suffix if $suffix;  ## add suffix for bridges
  }
  return $return_hash;
}


################################################################################
####                                                                           #
####  print_match -- print RTP where an n_byte word matches the next word     #
####                 (generated by compare(...) qv)                            #
####                                                                           #
```

```
##############################################################################

sub print_match {
  my $fill_ref = shift;
  my $line = shift;
  my $word_size = shift;
  # my $highest_rtp = 99;

  for (my $walk = 0; $walk <= (length($line) - 2*$word_size);
       $walk += $word_size) {

    my $word = ($walk/$word_size)+1;
    my $measure = POSIX::ceil($walk/$measure_size) + $initial_offset;
    my $printed_word = $print_all_words;
    my $print_line = sprintf("{%04d-%03d} ", $word, $measure);
    my $this_w = $fill_ref->{$walk};
    my $highest_score = 0;
    my $highest_rtp = ",00";
    for (my $rtp = 1; $rtp <= $bc; $rtp++) {
      my $rtp_s = defined($this_w->[$rtp]) ? $this_w->[$rtp] : 0;
      if ($rtp_s > $highest_score && $rtp_s > $0) {
            $highest_score = $rtp_s;
            $highest_rtp = sprintf(",%02d",$rtp);
      } elsif ($rtp_s == $highest_score && $rtp_s > 0) {$highest_rtp .= sprintf(",%02d",$rtp)}
     if ($rtp_s >= $minimum_score) {
        unless ($print_score) {$print_line .= sprintf("%02d ", $rtp)}
        else                  {$print_line .= sprintf("%02d-%02d ", $rtp, $this_w->[$rtp])}
        $printed_word = 1;
      } else {
        unless ($print_score) {$print_line .= " " x 3}
        else                  {$print_line .= " " x 6}
      }
    }
    $print_line .= defined($this_w->[$bc+1]) ? $this_w->[$bc+1] : ' ';
    printf("%s {%3d--%s}\n", $print_line, $highest_score, $highest_rtp) if $printed_word;
  }
}
```

Appendix 3: adowa.score

```
######################################################################
####                                                                 #
####  Score file for adowa.pl                                        #
####                                                                 #
######################################################################

-s      # print a summary of this file
-w12    # print a summary of every (X) matching beats
-g0     # initial gap size
-b12    # number of RTP/beat classes
-ms40   # minimum score to be printed
-a      # all lines will be printed
-hs     # print highest score

### Title

-t "Anku: Structural Set Analysis of African Music 1: Adowa"
-rf "anku_conc.txt"
-m6
-o5

#-t "Kaminsky: Transcription of Adowa Lead Drum"
#-rf "kam_conc.txt"
#-m12
#-o0

#-t "Kongo Zabana: African Drum Music: Adowa"
#-rf "zab_conc.txt"
#-m12
#-o2

#-t "Anku: Adowa Transcription found in Dissertation"
#-rf "anku_diss.txt"
```

```
#-m6     # beats in a measure
#-o8     # first encoded RTP(downbeat) falls on measure (X)


### Comparison 0

-c0 word  6
-c0 gap   6
-c0 score 30
-c0 rfb_attack 1
-c0 rp_silence 1
-c0 tone_insensitive 1

### Comparison 1

-c1 copy 0
-c1 score 15
-c1 walk_offset 12

### Comparison 2

-c2 copy 0
-c2 score 10
-c2 walk_offset -12

### Comparison 3

-c3 copy 0
-c3 score 10
-c3 rp_silence 0

### Comparison 4

-c4 word  12
-c4 gap   0
-c4 score 40
-c4 rfb_attack 1
-c4 rp_silence 1
-c4 tone_insensitive 1

### Comparison 5

-c5 copy 4
-c5 score 20
-c5 walk_offset 12

### Comparison 6 (Repeat of 0-5, but tonally sensitive; worth 1/2)

-c6  copy 0
-c7  copy 1
-c8  copy 2
-c9  copy 3
-c10 copy 4
-c11 copy 5

-c6  score 10
-c7  score 5
-c8  score 5
-c9  score 5
-c10 score 20
-c11 score 10

-c6  tone_insensitive 0
-c7  tone_insensitive 0
-c8  tone_insensitive 0
-c9  tone_insensitive 0
-c10 tone_insensitive 0
-c11 tone_insensitive 0

### Comparison 12 (to cover 24 beat patterns)

-c12 word 12
-c12 gap  12
-c12 score 40
-c12 rfb_attack 1
-c12 rp_silence 1
```

```
### Comparison 13 (internal symmetry)

-c13 word 6
-c13 gap 0
-c13 score 20
-c13 rfb_attack 1
-c13 rp_silence 1

### Comparison 14 (internal symmetry -- tonally insensitive)

-c14 copy 13
-c14 tone_insensitive 1

### Comparison 15 (5 points just for being a possible RTP; that is, one attacked beat after
###                              silence )

-c15 word 1
-c15 gap -1
-c15 score 5
-c15 rfb_attack 1
-c15 rp_silence 1
-c15 step_size 12

#### Comparison 16-22 (an extra 2 points for each beat of silence preceding a possible RTP)

-c16 word 1
-c16 gap -1
-c16 score 3
-c16 rp_silence 2
-c16 rfb_attack 1
-c16 step_size 12

-c17 copy 16
-c18 copy 16
-c19 copy 16
-c20 copy 16
-c21 copy 16
-c22 copy 16

-c17 rp_silence 3
-c18 rp_silence 4
-c19 rp_silence 5
-c20 rp_silence 6
-c21 rp_silence 7
-c22 rp_silence 8
-c22 score 5

#### Compare 23 (identify idiomatic bridges)

-c23 word 9
-c23 gap 0
-c23 score -50
-c23 suffix i
-c23 rp_silence 1
-c23 step_size 12

-c24 copy 23
-c24 gap 9
```

Appendix 4: A sample output from rtp.pl running adowa.score

```
** Anku: Structural Set Analysis of African Music 1: Adowa **

All phrases will be printed, even ones with no matches.
Measure Size  :  6, First Measure:  5
Minimum Score : 40
```

beginning				*highest*	*highest*
set	*m.*	*strongly heard RTPs*	*bridge = i*	*score*	*RTP(s)*
{0001-005}				{ 15--,02,09}	

{0002-007}		02							09			{158--,09}
{0003-009}		02							09	11		{185--,09}
{0004-011}									09	11		{125--,09}
{0005-013}			03						09			{135--,09}
{0006-015}		02							09			{138--,09}
{0007-017}		02							09	11		{155--,09}
{0008-019}		02							09	11		{100--,09}
{0009-021}												{ 31--,02}
{0010-023}												{ 0--,00}
{0011-025}			03								i	{ 45--,03}
{0012-027}												{ 8--,03,12}
{0013-029}											i	{ 15--,02,06}
{0014-031}		02				06						{141--,02}
{0015-033}		02				06		08		11		{186--,02}
{0016-035}		02						08		11		{168--,08}
{0017-037}		02						08		11		{175--,11}
{0018-039}								08		11		{110--,11}
{0019-041}				04				08				{ 63--,08}
{0020-043}	01			04								{ 65--,04}
{0021-045}												{ 8--,04,10}
{0022-047}												{ 15--,02,08}
{0023-049}		02						08				{129--,08}
{0024-051}								08		11		{ 60--,11}
{0025-053}												{ 25--,11}
{0026-055}											i	{ 5--,03}
{0027-057}												{ 8--,03,12}
{0028-059}												{ 17--,07}
{0029-061}				04			07					{ 68--,07}
{0030-063}												{ 23--,07}
{0031-065}				04			07					{ 68--,07}
{0032-067}												{ 23--,07}
{0033-069}				04			07					{ 45--,04,07}
{0034-071}												{ 23--,07}
{0035-073}				04								{ 45--,04}
{0036-075}												{ 23--,07}
{0037-077}												{ 5--,04,07,09,11}
{0038-079}												{ 23--,07}
{0039-081}			03						09			{143--,09}
{0040-083}									09			{ 55--,09}
{0041-085}												{ 11--,07}
{0042-087}												{ 28--,08}
{0043-089}												{ 23--,07}
{0044-091}											i	{ 5--,03}
{0045-093}												{ 8--,03,12}
{0046-095}												{ 11--,09}
{0047-097}												{ 8--,08}
{0048-099}												{ 28--,05}
{0049-101}												{ 11--,01}
{0050-103}												{ 8--,05}
{0051-105}												{ 8--,08}
{0052-107}												{ 28--,05}
{0053-109}												{ 11--,01}
{0054-111}												{ 10--,08}
{0055-113}								08				{ 63--,08}
{0056-115}		02				06		08				{158--,08}
{0057-117}						06		08		11		{115--,11}
{0058-119}										11		{ 80--,11}
{0059-121}		02								11		{ 88--,02}
{0060-123}		02			05					11		{147--,05}
{0061-125}					05					11		{170--,11}
{0062-127}										11		{ 55--,11}
{0063-129}							07		09	11		{ 45--,07,09,11}
{0064-131}												{ 28--,05}
{0065-133}												{ 5--,07,09,11}
{0066-135}												{ 28--,05}
{0067-137}											i	{ 5--,03,11}
{0068-139}												{ 15--,02,05}
{0069-141}		02			05							{138--,05}
{0070-143}					05					11		{ 79--,05}
{0071-145}			03		05					11		{140--,11}
{0072-147}			03		05					11		{ 90--,03}
{0073-149}			03									{105--,03}
{0074-151}										11		{ 66--,11}
{0075-153}					05							{ 69--,05}
{0076-155}												{ 30--,05}
{0077-157}			03		05							{ 70--,03}

```
{0078-159}       03                              { 65--,03}
{0079-161}                               11      { 66--,11}
{0080-163}                                     i { 31--,11}
{0081-165}                            10    12   { 48--,12}
{0082-167}                                     i {  5--,03}
{0083-169}                                       { 15--,02,05}
{0084-171}    02       05                        {153--,05}
{0085-173}    02       05                11      {179--,05}
{0086-175}             05                11      {125--,11}
{0087-177}       03    05                11      {195--,11}
{0088-179}       03    05                11      {176--,11}
{0089-181}                               11      { 61--,11}
{0090-183}                                     i {  5--,03}
{0091-185}                            10    12   { 48--,12}
{0092-187}                                     i {  5--,03}
{0093-189}                                       {  8--,03,12}
{0094-191}                                       { 28--,01}
{0095-193}       03          07       10         { 45--,03,07,10}
{0096-195}                                       { 14--,01}
{0097-197}       03          07                  { 45--,03,07}
{0098-199}                                       { 14--,01}
{0099-201}                                       {  5--,03,07,10}
{0100-203}                                       { 20--,06}
{0101-205}                06                12   {172--,12}
{0102-207}                06                12   {158--,06}
{0103-209}                06             11      {194--,06}
{0104-211}                06             11      {134--,06}
{0105-213}                06                     {129--,06}
{0106-215}                                  12   { 48--,12}
{0107-217}       03                            i { 45--,03}
{0108-219}                                       {  8--,03,12}
{0109-221}                                     i {  5--,03}
{0110-223}                                       {  8--,03,12}
{0111-225} 01                                    { 68--,01}
{0112-227}                   07          11      { 81--,07}
{0113-229} 01       04       07          11      {161--,07}
{0114-231} 01       04       07          11      {116--,07}
{0115-233}                               11      { 80--,11}
{0116-235}                                       { 20--,11}
{0117-237}                            10         { 45--,10}
{0118-239}                                       { 25--,10}
{0119-241}                                       { 17--,01}
{0120-243}                                       {  8--,01,04}
{0121-245}                                       { 26--,07}
{0122-247}                   07                  { 60--,07}
{0123-249}                                       { 31--,07}
{0124-251}                                       { 14--,11}
{0125-253}                                       { 11--,08}
```

Appendix 5: set_density.pl

```
#!/usr/local/bin/perl -w

### set density -- counts number of attacks within each n note set

my $set_size = 12;
my $previous_set;

foreach (<STDIN>) {
  for (my $walk = 0; $walk < (length($_) - $set_size); $walk += $set_size) {
     my $examined_set = substr($_, $walk, $set_size);
     next if $examined_set eq $previous_set;
     $previous_set = $examined_set;
     my $stars = $examined_set =~ tr/[1-9]/[1-9]/;
     my $star_print = "*" x $stars;
     printf("%3d | %s\n", ($walk/$set_size), $star_print);
  }
}
```

(The output of set_density.pl is shown as a table within the text)

Appendix 6: prime_forms.pl

```
#!/usr/local/bin/perl -w

### prime forms -- returns the prime forms of each n note set
###                in a way that allows them to be sorted

my $set_size = 12;
my $previous_set = "";


print "attacks |  normal form | position | original form\n";
foreach (<STDIN>) {
  for (my $walk = 0; $walk < (length($_) - $set_size); $walk += $set_size) {
    my $examined_set = substr($_, $walk, $set_size);
    # next if $examined_set eq $previous_set;
    $previous_set = $examined_set;

    #### eliminate pitch distinctions

    $examined_set =~ s/0/o/g;
    $examined_set =~ s/\d/x/g;

    # print $examined_set, "\n";
    my $set_form = "";
    my $current_interval = 0;
    my $initial_rests = 0;
    my $attacks = 0;
    for (my $set_walk = 0; $set_walk < $set_size; $set_walk++) {
      my $current_attack = substr($examined_set, $set_walk, 1);
      if ($set_walk != 0 && $current_attack eq "x") {
        if ($attacks) {
          $current_interval = one_digit($current_interval);
          $set_form .= $current_interval;
          $current_interval = 1;
          $attacks++;
          # print "$set_form\n";
        } else {  ## tack initial rests to end of set
          $initial_rests = $current_interval;
          $current_interval = 1;
          $attacks++;
          # print "inital_rests: $initial_rests ... $set_form\n";
        }
      } elsif ($set_walk == 0 && $current_attack eq "x") {
        $attacks++;
        $current_interval++;
      } else {
        $current_interval++;
      }
    }
    my $final_interval = $current_interval + $initial_rests;
    $final_interval = one_digit($final_interval);
    $set_form .= $final_interval;
    my $normal_set = find_normal_form($set_form);
    my $position = ($walk/$set_size)+1;
    if ($attacks == 0) { $normal_set = "-"; $set_form = "-";}
    printf("     %2s | %12s |      %3s |  %12s \n",
      $attacks, $normal_set, $position, $set_form);

  }
}


sub one_digit {
  my $line = shift;
  if    ($line < 10) {return $line}
  elsif ($line == 10) {return "A"}
  elsif ($line == 11) {return "B"}
  elsif ($line == 12) {return "C"}
  elsif ($line == 13) {return "D"}
  elsif ($line == 14) {return "E"}
  elsif ($line == 15) {return "F"}
  elsif ($line == 16) {return "G"}
```

```
  else {return "*"}
}

sub find_normal_form {
  my $set = shift;
  my @set_rotation;
  for (my $i = 0; $i < length($set); $i++) {
    $set_rotation[$i] = substr($set, $i) . substr($set, 0, $i);
    # print "$i $set_rotation[$i]\n";
  }
  my @sort_set = sort @set_rotation;
  my $normal_form = $sort_set[0];
  # print "sorted: $normal_form\n";
  return $normal_form;
}
```

Appendix 7: output of prime_forms.pl for Anku 1992

Bold indicates normal (prime) forms appearing in more than one form in the piece .

attacks	normal form	position	original form
0	-	1	-
1	C	94	C
1	C	111	C
2	1B	30	1B
2	1B	34	1B
2	1B	38	1B
2	1B	43	1B
2	1B	126	1B
3	147	32	147
3	147	36	147
3	156	28	615
4	1119	48	1119
4	1119	52	1119
4	1119	64	1119
4	1119	66	1119
4	**1218**	**29**	**1218**
4	**1218**	**31**	**1218**
4	**1218**	**69**	**8121**
4	**1218**	**84**	**8121**
4	1272	96	2127
4	1272	98	2127
4	1272	100	2127
4	1362	56	2136
4	1524	72	1524
4	1524	80	1524
4	1524	88	1524
4	1524	89	1524
4	2226	44	2226
4	2226	90	2226
4	2433	112	2433
4	2433	114	2433
4	2433	115	2433
4	2433	123	2433
5	**11145**	**2**	**11145**
5	**11145**	**103**	**51114**
5	**11145**	**104**	**51114**
5	**11145**	**105**	**51114**
5	11217	35	11217
5	11217	42	11217
5	**11415**	**23**	**51141**
5	**11415**	**41**	**11415**
5	**11415**	**74**	**15114**
5	**11415**	**79**	**15114**
5	11433	113	11433
5	12126	119	12126
5	13134	124	13134
5	14232	47	23214
5	14232	49	23214
5	14232	51	23214
5	14232	53	23214
5	14232	55	23214

5	**15222**	**63**	**22215**
5	**15222**	**65**	**22215**
5	**15222**	**71**	**21522**
5	**15222**	**77**	**21522**
5	**15222**	**87**	**21522**
5	**22224**	**11**	**42222**
5	**22224**	**13**	**42222**
5	**22224**	**14**	**22242**
5	**22224**	**26**	**42222**
5	**22224**	**82**	**42222**
5	**22224**	**92**	**42222**
5	**22224**	**107**	**22224**
5	**22224**	**109**	**42222**
5	22233	12	22233
5	22233	27	22233
5	22233	45	22233
5	22233	68	22233
5	22233	81	22233
5	22233	83	22233
5	22233	91	22233
5	22233	93	22233
5	22233	108	22233
5	22233	110	22233
5	22233	120	22233
6	111126	60	261111
6	111126	61	261111
6	111126	62	261111
6	111144	125	411114
6	111315	39	111315
6	111315	75	151113
6	112125	20	511212
6	113223	25	231132
6	114123	15	231141
6	114123	16	231141
6	114123	17	231141
6	114123	18	231141
6	114123	24	231141
6	114312	116	114312
6	114312	121	114312
6	121215	70	215121
6	121215	85	215121
6	121215	86	215121
6	121233	122	121233
6	122124	73	122124
6	122124	78	122124
6	122232	58	232122
6	123123	21	312312
6	123213	57	232131
6	123213	59	232131
6	222222	67	222222
7	1111422	46	4221111
7	1114113	6	1311141
7	1114122	3	2211141
7	1114122	4	2211141
7	1114122	7	2211141
7	1114122	8	2211141
7	1114122	9	2211141
7	1121223	19	2311212
7	1121232	117	1212321
7	1123122	118	2112312
7	1212222	33	2221212
7	1212222	37	2221212
8	11131113	101	11131113
8	11131113	102	11131113
8	11131113	106	11131113
8	11131122	5	22111311
8	11211222	22	21121122
8	11221212	95	12121122
8	11221212	97	12121122
8	11221212	99	12121122
9	111121113	40	112111311
10	1111111113	50	1111131111
10	1111111113	54	1111131111
10	1111111212	76	2121111111
12	111111111111	10	111111111111