TIPPING THE ICEBERG: MISSING ITALIAN POLYPHONY FROM THE AGE OF SCHISM

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Describing a collection of music is impossible without understanding its extent. It takes seeing the repertory as a whole to distill its salient features, its internal subdivisions, and, above all, the distinctive exceptions that invigorate music and inspire composers. But we doubt our ability to get a handle on a repertory when it is distant or largely incomplete. In these cases our perspective is obscured, our understanding partial. Our conclusions are subject to revision. They are, in short, inconclusive.

We would be more assured about our work if we were convinced that we lacked only a little from the repertory being studied. But Medieval and Renaissance musicology is particularly stymied by its anxiety over how much material has been lost. We are warned repeatedly that we possess only the tip of the iceberg and therefore are incapable of seeing the reality of the past.[1]

I wish to acknowledge most particularly Lisa Friedland (Department of Computer Science, University of Massachusetts, Amherst) for conversations and advice which resulted in many of the mathematical models used in this paper, David Tabak (National Economic Research Associates) for first noting the similarities to animal population sampling methods, and, for discussions on specific statistical techniques used in this project, Shahar Boneh (Department of Mathematical and Computer Sciences, Metropolitan State College of Denver) and William Bossert (Department of Biophysics, Harvard University). On the humanities side, I am grateful for conversations with and comments from Albert Ascoli, Margaret Bent, Sean Gallagher, Oliver Huck, Thomas Forrest Kelly, John Nádas, and Agostino Ziino. Earlier versions of this paper were read in 2005 at Kalamazoo and Tours.

1. The specific metaphor of "the tip of the iceberg" for surviving music appears, *inter alios*, in Godwin, "'Main Divers Acors'," 159, regarding the surviving music of the repertory discussed in this paper; Dean, "Evolution of a Canon," 151, referring to surviving music sung at the papal chapel; Flack, Letter, 169, with respect to surviving round canons (replying to a suggestion about unwritten repertories made by Ernest H. Sanders, similar in spirit if not in vocabulary); Stevens, "Musique d'orgue," 141, regarding the proportion of surviving music from pre-Reformation England; Robertson, "Benedicamus Domino," 14, as a rhetorical question about the size of the unwritten tradition; Litterick, "Italian instrumental ensemble music," 129, referring to the relative numbers of notated vs. unwritten texts; Pirrotta, "Oral and Written Traditions," 72, describing the relationship between written and unwritten music (his statement has been quoted in Petrobelli, "Pirrotta," *xiv*, and Treitler, *Voice and Pen*, 11); Zazlaw, "Review," 43, comparing known to unknown composers. I believe, but cannot prove, that this list represents less than the tip of the iceberg of the total number of iceberg metaphors used to describe lost Medieval and Renaissance music.

Musica Disciplina 54, 2009.

The idea that our losses are vast has pervaded medieval music to such an extent that it is both undisputed and rarely discussed. Yet it is necessary to distinguish at least three different definitions of lost medieval music, three definitions that are too often lumped together. First are the unwritten traditions of music—either improvised or fixed but transmitted only or mainly in sound. Second are the lost musical manuscripts known only through library records, fragmentary remains, or gaps in constructed stemma. Finally, there are the pieces that once were written down but have been lost through the ages.

I distinguish among these three categories because I have little to say about the first two definitions of missing music, and even that consists mostly of hunches. However, there are many substantive ways of investigating the last category: lost pieces that were once copied into now-missing music manuscripts. These methods of investigation have up-to-now been unexplored, but, if followed, lead to a striking change in our view of medieval repertories. Instead of the thinnest edge of a vast repertory of missing pieces, the surviving body of Italian music from the period of the Great Papal Schism represents a large portion of its original written extent.

The many ways of considering the size of the lost repertory of late Trecento and early Quattrocento polyphony all lead to the conclusion that the metaphor of the iceberg is a bad one. The methods bring in evidence from poetry, from indices of lost manuscripts, from a computer-based statistical model, and from recent discoveries of fragments. Since these methods have wide applicability beyond the early fifteenth-century, the paper will end by suggesting their utility to other branches of musical scholarship and humanistic studies. But first, the literary evidence.

The Lost Songs of Prodenzani and Sacchetti

Simone de' Prodenzani (ca. 1355–ca. 1440) cannot, by any stretch of the imagination, be called one of Italy's greatest poets. Though they are entertaining, his sonnets are often facile treatments of popular subjects or of splendid evenings. Ironically, their largest deficiency as poetry becomes the greatest utility for this study: his verse includes many long lists, especially lists of titles of songs and dances.

Within these lists, Prodenzani's citations of musical compositions can be identified with reasonable certainty. Some of the pieces he cites are unknown; they are among the "semi-lost," that is, pieces whose music and poetry are not available today, but whose one-time existence is documented. Yet these semi-lost pieces are in no way the overwhelming norm among

Table 1. Simone de Prodenzani, "Il Saporetto," Sonnet 35.

	Known works	Unknown
COLLA VIVOLA FE' Cançon di Maio		
Rosetta che non cambi mai colore,	(1)	
Ie sui nafres tam fort, *Dolce sapore,*	(2)	(1)?
Comme da te partir me degio oma'io?		(2)?
D'amor languire e puoi *El dolce raio,*	(3)	(3)?
O rosa bella, che m'alegrie 'l core,	(4)	
Legiadra donna e poi ***Donna d'amore,***	(5) (6)	
Un fior gientile del qual mi 'namoraio,	(7)	
Questa mirabil donna, Margarita,		(4)?
Con lagrime bagniando el suo bel viso,	(8)	
Deducto se' e fé *Sella mia vita,*	(9)	(5)
Costei sarebbe bella in Paradiso,		(6)
Non credo, donna, *O giemme incolorita* del Cicognia una parte fu l'aviso.	(10)	(7)

Prodenzani's references, as an analysis of the thirty-fifth sonnet of *Il Saporetto* demonstrates (Table 1).[2]

Of the works or possible works cited, copies of the ten shown in bold in Table 1 are extant. By the most conservative count, 10 of the 17 pieces mentioned in the sonnet survive (59%). But it is possible that the survival rate is even higher. John Nádas has equated "El dolce Raio" with Ciconia's *Le Ray au Soleyl,* and has also tentatively connected "Questa mirabil donna, Margarita" with the refrain of the ballade *N'a pas longtemps* which discusses the very pleasing and beautiful Margarite.[3] Nádas, like Carboni, does not consider "dolce sapore" a musical work. If each of these suppositions be true,

2. Edition adapted from Carboni, *Prodenzani,* vol. 2, 260–63. Compare with Reale, *Sollazzo e Saporetto,* 147, and the significantly older edition, Debenedetti, *Sollazzo e il Saporetto,* 110. I agree with Reale and Carboni that the "canzon' di maggio" describe the remaining songs, rather than being the title of a song itself; I agree with Debenedetti and Reale that "Dolce sapore" is more likely the title of a song than a description of the songs around it. Furthermore since few pieces in *Il Saporetto* are cited by only a single word, I have considered "El dolce raio" (rather than simply, "Raio") to be an incipit, and thus classify it as a lost work. Finally, the whole of line two, and not just the first word, is the incipit to Zachara's *Rosetta.*

3. Nádas, "Cautious reading," 35. The quotation in *N'a pas longtemps* is "La très plaisant et belle Margarite," so it is not an exact match. Reale considers "Margarite" a reference to *Margarite, fleur de valeur* by Binchois; this is not completely impossible on chronological grounds, but is highly unlikely, given the rest of Prodenzani's repertory. David Fallows, "Ciconia's last songs," 114, summarizes the arguments that suggest that *Le Ray au Soleyl* should no longer be regarded as an "opus dubium." "Come partir da te me debbo mai," a quotation from Boccaccio's *Filostrato* (III, 44–45), survives as a textual incipit in a poem on a fragment in the Bologna Archivio di Stato; see Antonelli, "Tracce," 30–31. *Se la mia vita* is a lost ballata by Giovanni, son of Gherardello da Firenze.

as many as 12 of 16 mentioned pieces survive (75%). Neither of these figures is a tip-of-the-iceberg number.

A look at the pieces mentioned in the rest of the poems in *Il Saporetto* shows that sonnet thirty-five is representative of the repertory as a whole. Prodenzani cites by name 84 works or genres in *Il Saporetto*, of which 59 seem to be individual songs or motets.[4] Of the songs, Nádas finds concordances for 40, with three possibly to be identified with surviving pieces, leaving 16 as missing. If we exclude the three attributions that Nádas labels with a question mark, there is a 71% survival rate.[5] I might suggest an even more cautious reading that questions four more of Nádas' identifications (*Or sus vous dormez trop, En mon cuer, Che ama ne la lingua*, and (as before) *Le ray au soleyl)*. But even after omitting these four, there is still a 65% survival rate for the songs and motets. Whether we take 71% or 65% (or some number in between) as the actual percentage, the conclusion remains the same: the majority of the songs that were sung or played in *Il Saporetto* are ones that can still be performed today.

Another poet has given us a glimpse of a lost musical world, this time listing settings of his own lyric poetry. Franco Sacchetti prepared several editions of his works, and, in later editions, he was careful to note which of his poems had been set to music and by whom. Table 2 gives as an example the works set by Niccolò da Perugia, the composer with the most settings.

Over half of Niccolò's twelve works on texts by Sacchetti survive today (7 of 12, or 58%). Some other composers fare less well: as a result, the overall survival rate for all the texts that Sacchetti records as being set is 12 of 34, or 35%. Gallo takes this figure as indicative of "how large a propor-

4. These numbers are based on Nádas, "Cautious reading," 34–36. To avoid biasing the results, except when explicitly noted, I will be using Nádas's identifications of pieces and concordances even in the few cases where I disagree with them. Of the 25 pieces and genres that are not individual songs, three are liturgical compositions that cannot be traced to a particular setting, eight are titles of song genres (e.g., calate de maritime et campagnia, cançon de Lombardia et de Romagnia, del Çacchara suoi caccie et suoi cançone), and fourteen are dances or dance pairs identifiable either by mention of dancing or titles in the form *La [X]* (e.g., *La picchina, La Forosetta, La Marinella, La casa bassa, Rigolecto* [which is a genre rather than a specific dance], *L'alvadança e l'trotto*); these dances represent music that was normally not written at all, and thus were probably not lost over the centuries. (See Nosow, "Dancing.")

5. If all three of the ascriptions questioned by Nádas are correct, we would have a 73% survival rate; if they are wrong in every case, the survival rate would instead be 68%, a difference of only 5%. Thus a recurring trope of this paper will be that small errors in identification do little to shake the overall premise that the surviving repertory represents a substantial portion of what once existed.

Table 2. Niccolò's Works Mentioned in Sacchetti's Catalog.

Come selvaggia fera fra le fronde (M)
Come la gru quando per l'aere vola (M)
Correndo giù del monte a le chiar'onde (M)
Di diavol vecchia femmina ha natura (B)
Nel mezzo già del mar la navicella (M)
Passando con pensiero per un boschetto (Ca)
Una augelletta, Amor, di penna nera (M)
Chi 'l ben sofrir non pò (B)
Povero pelegrin salito al monte (M)
Lasso, s'io fu' già preso (B)
State su, donne!—Che debian noi fare (Ca)
Chi vide più bel nero (B)

M = madrigal, B = Ballata, Ca = Caccia. Works which survive today are shown in bold type.

tion of fourteenth-century music has been lost."[6] However, the evidence is really not so dismal. Not only is 35% far more than the percentage of visibility suggested by the ubiquitous iceberg metaphor, but also the numbers improve substantially in the music of the last third of the Trecento, the period from which the vast majority of our surviving fourteenth-century Italian manuscripts stem. Of the eleven poems that Sacchetti records as being set in or after 1365, six survive (55%), and two of the lost settings were by Sacchetti himself and thus possibly never copied into manuscripts of polyphonic music. So of the later Trecento music composed by professional composers, six of nine survive, or 67%, or approximately the same proportion as of the coeval repertory described by Prodenzani.

Concordance Rates and Repertory Estimates

Up to this point, my estimates of the size of the missing repertory have been based on citations of lost works. But even the works that do survive can speak volumes about the extent of our losses. Scholars expect that there would be fewer lost pieces among the repertories with few *unica* than among those with more. This is precisely the intuition David Fallows invoked when he wrote that "although music of Binchois appears in over 50 manuscripts of the 15th century, the survival of many pieces in only one source implies a substantial loss over the centuries."[7] We can apply this notion that the

6. Gallo, *Music of the Middle Ages*, 65–66, from which Table 2 is drawn.
7. Fallows, "Binchois," 581.

number of surviving copies gives an idea of the rate of survival of a composer's works to the genres of music composed or copied during the Great Schism.

Table 3 separates by genre and number of sources the Italian pieces found in at least one manuscript from the era of the Schism. In addition, pieces ascribed to composers who worked during that period are included even if they only appear in later sources (*e.g.*, the motets of Ciconia found only in *Bologna Q15*).[8] Older repertory that is found only in older sources, such as the *Rossi* unica, are not included, because there simply is not enough repertory from the earlier period (measured in numbers of either manuscripts or pieces) to allow us to make sound estimates. Excluded also are works of simpler polyphony (non-mensural or mensural pieces with fewer than four different rhythmic levels; for instance, harmonized Credo Cardinalis settings), contrafacts where the secular version survives (although probable contrafacta such as the Kyrie "Rondello," based on unknown models, are included among the liturgical works), and anonymous pieces in Italian manuscripts that are in the "international repertory," which I define as works in six or more manuscripts of which over half are not Italian (*e.g.*, Gloria "Qui sonitu melodie"). Also excluded are manuscripts discovered in and after 2004; the reasons for this last exclusion will be given later in the paper.[9]

A surprising revelation of Table 3 is that there are nearly as many surviving Latin works as there are madrigals, often considered the most quintessentially Italian of all Trecento genres. But the much higher rate of survival in single sources for the Latin compositions suggests that many more sacred works are missing than are madrigals. And our manuscript discoveries bear witness to this fact: of the twenty-three sources announced or discovered since *San Lorenzo 2211* in 1984, only the fragment *Trent 60*

8. See the list of "Manuscript sigla" at the end of the article for all italicized names of sources.

9. For an expanded version of the table with counts for each manuscript given separately, see Cuthbert, "Trecento Fragments," 57–60. The inventory is based on an unpublished inventory of all sources 1350–1420 in preparation by the author. The most accurate publically available inventory of secular Trecento music is online, Huck, "Musik des frühen Trecento." Differences between the number of concordances listed in Huck's inventory and in this table are largely due to my inclusion of instrumental arrangements of secular work. (However, instrumental diminutions that are not arrangements of vocal music are not included in this table). Huck's database excludes all keyboard repertory; this difference particularly affects the counts for Ciconia's *Con lagrime*. The category "cacce" includes other canonic works such as canonic madrigals. Liturgical music primarily comprises settings of the Mass Ordinary and Benedicamus Domino. The "Non-Liturgical Latin" category encompasses both sacred and ceremonial motets.

Table 3. Copies of Italian Music ca. 1380–1420.

	No. of Pieces				
No. of Sources	Cacce	Madrigals	Ballate	Liturgical	Non-Liturgical Latin
Eight		3	1		
Seven		1	3		
Six	1	4	7	1	
Five	1	15	9	3	
Four	2	15	17	1	
Three	2	21	47	5	3
Two	7	33	83	15	8
One	14	74	242	85	36
Totals	27	166	409	112	47
Unica %	52%	45%	59%	76%	77%
Total Copies	53	393	732	169	61

and the newest discovery, the *Manganelli Fragment*, contain new madrigals, and the only new cacce are found in *Trent 60* and a manuscript earlier than the period this study concerns, the *Mischiati Fragment*.[10] By contrast, twelve Italian sources found since 1984 contain previously unknown compositions in Latin. We will return later to the most recent sources, those announced post-2003.

As noted above, based on the percentage of *unica* there should be a higher percentage of missing Latin works than of missing works in the vernacular. It is possible to go further and actually estimate the number of lost works. There are several different statistical techniques that can be used; this section will use three. Because the methodologies and assumptions differ in each of these techniques, each technique gives somewhat different results; nonetheless, none of them give results similar to the iceberg hypothesis. (Readers uninterested in the math behind the methodologies may want to skip to Table 4.)

The first two models were adapted from those commonly used in ecological studies to calculate the number of different species in a population where it is impossible or not feasible to capture every member. An

10. The discovery of the *Manganelli Fragment* was announced at the conference "Beyond 50 years of Ars Nova Studies at Certaldo." The *Mischiati Fragment* was recently described in Gozzi and Ziino, "Mischiati Fragment." Gozzi, "Nuovo Frammento," 251, classifies the fragmentary...*chi cava 'l morso* in *Trent 60* as a caccia, which the text and form support, but since neither Gozzi nor I have been able to find a satisfactory canonic solution, it may be a madrigal.

example of such a question would be: "How many different species of fish live in a certain lake?" Bradley Efron and Ronald Thisted (hereafter E & T) adapted these techniques to create a model for estimating the number of words that do not appear in Shakespeare's writings, but would likely appear if many additional plays by the Bard suddenly turned up.[11] They derived a formula for the number of new discoveries if the corpus were doubled—that is, if twice as many works by Shakespeare were found. The same technique would work if the corpus of musical sources was doubled and 732 more copies of late-Trecento ballate were found. The formula estimates the number of new discoveries (Δ) based on the number of works with one copy (n_1), the number of works with two copies (n_2), and so on:

$$\Delta = n_1 - n_2 + n_3 - n_4 + \ldots$$

Using the numbers of Trecento ballate presented in Table 3, their formula estimates that if we were to double the number of manuscripts we currently have, we would find 193 new ballate:

$$\Delta = 242 - 83 + 47 - 17 + 9 - 7 + 3 - 1 = 193$$

The E & T method was criticized and refined in an article that uses a slightly different approach tending to give lower estimates despite the fact that this model estimates the *total* number of lost "species" (and not just the number we would expect to find if the size of the corpus doubled). The Boneh, Boneh, and Caron (hereafter BBC) approach also begins with a formula that is almost as simple as the previous:[12]

$$U_0 = n_1 e^{-1} + n_2 e^{-2} + n_3 e^{-3} + \ldots$$

However, since this formula introduces some bias, it can be corrected by solving (with a computer) for U in the following equation:

$$U\,(1 - e^{-n_1/U}) = U_0$$

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11. Efron and Thisted, "Estimating the number," 437. In addition to the method that will be used in this paper, Efron and Thisted also suggest a way of estimating the *total* number of words Shakespeare knew. Since that technique is strongly criticized in Bohen, Bohen, and Caron, "Estimating the Prediction Function," only Efron and Thisted's first estimation formula will be used here. A non-technical summary of Efron and Thisted's article appears in Bennett, Briggs, and Triola, *Statistical Reasoning*, available on-line.

12. Bohen, Bohen, and Caron, "Estimating the Prediction Function," 375. Their variable names have been changed slightly in this paper to accord with those used by Efron and Thisted. e represents the base of the natural logarithm, or approximately 2.7.

Table 4. Some population estimates using different models.

	Surviving	Three Models for Number of Lost Works		
		E & T *	BBC and survival rate	Cuthbert and survival rate
Cacce	27	7	7 (78%)	4 (87%)
Madrigals	166	56	39 (81%)	10 (94%)
Ballate	409	193	118 (78%)	98 (81%)
Liturgical Pieces	112	76	37 (75%)	78 (59%)
Non-Liturgical Latin Works	47	31	16 (75%)	58 (45%)
Totals	**761**	**363**	**217 (78%)**	**248 (75%)**

* N.B., E & T is an estimate of the number of new pieces we would find if the size of the corpus were doubled. Thus, the overall survival rates cannot be calculated.

For instance, for the late-Trecento ballate, the BBC approach estimates that there are 118 lost ballate: we thus would have 78% of all the ballate once written.

The appendix to this article sketches out a third method for estimating the number of lost pieces. It begins by assuming that all pieces were equally available and equally likely to be copied and then uses that assumption to predict how many pieces we would expect not to survive. (This assumption might seem invalid or even absurd: however, later in the appendix a method called cross-validation is used to test how closely the distribution of pieces in late Trecento manuscripts approximates random collecting; the difference is actually quite small).[13] This third model (Cuthbert) predicts even fewer lost ballate (98) while suggesting that a larger portion of sacred music no longer survives. Table 4 summarizes the results of all three models for each repertory.

Two obvious conclusions emerge from the calculations given in this table. First, that not all of the estimates accord perfectly well, especially in the prediction of the number of missing madrigals. But second and just as importantly, none of the models suggests a vast lost repertoire. In fact, the BBC and Cuthbert models predict survival rates within the same range as those seen in the literary evidence, mostly between sixty and eighty percent.

It is intuitive, but crucial to bear in mind, that none of the methods used in this paper (based on either statistical models or literary evidence) to

13. For a recent and controversial example of a process that is not at all random but yet is perfectly modeled by random behavior, see Hodgson, "Wine Competitions" (summarized in Mlodinow, "Hint of Hype"), which shows that previous success in wine competitions is no better than random coin flips in predicting which wines will win gold medals at future competitions. I thank John McKay for pointing out the connection to this work.

reach estimates of the size of a repertory can inform us about the size of other repertories not discussed here. The size of the Duecento repertory cannot be predicted from even the closest study of music of the late Trecento. Similarly, a discovery of a new repertory unrelated to the music that now survives would upset the predictions; for instance, the large collection of Cypriot polyphony in *Turin 9* could not have been predicted even if these models had been run on the other French sources. Nonetheless, the possible existence of musical repertories unhinted at by present-day manuscript evidence should not hinder the study of the surviving repertories, nor should it prevent us from forming conclusions untroubled by the illusionary shadow of an hidden iceberg of lost works.

One standard way to test statistical models is unfortunately rather difficult for us to employ: find new sources and see how they accord with the model. Obviously, new Trecento sources cannot be found whenever we want. However, since I first presented a paper on this project, in 2004, several new sources have been found. I have purposely not incorporated them into the studies above so that we may test how well new manuscripts accord with the predictions already proposed: few new secular pieces with more new sacred pieces. The fragments *Perugia Cialini* contain Mass movements, motets, and madrigals. All of the madrigals were previously known while all the Mass movements and most of the motets were new discoveries. A fragment in Siena, *Siena Ravi 3*, contains five Latin-texted works; three were known and two unknown. *London 82959* and *Houghton 420* each contain one polyphonic Mass movement: London's was known, Houghton's unknown.[14] Oliver Huck identified a work previously thought to be unique in *Seville 25* as a copy of *O Rosa Bella*,[*] thereby removing one *unicum* from our sources of secular music. The *Bologna Archivio Covers* contain one already known ballata, while the already-mentioned *Manganelli Fragment* is the exception, containing a previously unknown madrigal. The fact that the *Manganelli Fragment* does not perfectly conform to the model is not significant, for models deal in generalities and not specifics. But the overall trend of recent discoveries is that of the model: new sources bringing with them mostly old secular and new sacred music.

14. The Gloria in *Siena Ravi 3* can now be identified with two concordant sources in Franco/Spanish fragments. See Cuthbert and Nyikos, "Style, Locality." Reports on both the London and Houghton sources are forthcoming from different authors. Both contain monophonic and polyphonic music mixed in puzzling ways.

[*] For "O rosa bella" read "La bella stella."

Popularity and Transmission

The study and analysis of medieval music has always been, and will always be, a selective art. That is to say, some works and some composers are more studied than others, and this selection at best informs, or at worst skews, our view of the period being studied. Given the limited time and resources with which scholars work, they may wish to focus their efforts on those pieces which were most well-known or most popular in the period studied. Unfortunately determining which pieces were popular at the time they were written is a difficult task, sometimes seen as impossible.

We often think that a work in many sources must by definition have been popular—or at least, that, when the vast unwritten tradition is taken into account, we can at least say it was popular among those who copied and read music. There are similar metrics which determine the popularity of pieces today, including the number of performances or of record sales. Such measures are less predictive of popularity, however, when the number of sources is very low. For just as a random series of coin flips will occasionally have a long string of heads without having any meaning behind it, so too can a piece of music appear in many different manuscripts purely by the vagaries of preservation.

In the previous section, some of the models had the assumption that each piece was equally likely to be selected, implying that the underlying popularity of each was equal. Testing this hypothesis by the method of cross-validation showed that the model predicted results similar to the numbers that actually survive. But any deviation from the random model, however small, should be investigated, because these differences hold the key to identifying the truly popular polyphonic works during the Papal Schism.

How likely is it that a piece that is copied in, say, six sources, appears so often out of chance rather than because it was specifically popular? For example, *Tosto che l'alba* and *Usellet(t)o selvag(g)io* are found in five and six sources respectively. No other canonic works are found in more than four sources. We might therefore conclude that these were popular cacce. Yet, if all cacce were once equally popular, given the surviving manuscript situation, it is quite likely that at least one caccia would appear in five sources. Thus we cannot say without other testimonies that *Tosto che l'alba* was definitely a popular song for its time. However, there is only a 2% chance that, if all cacce were equally popular, any would appear in six sources. Thus it is very likely that the caccia-madrigal *Usellet(t)o selvag(g)io* was popular. But we cannot say anything definitive about the popularity of the two cacce

which appear in four sources, *Cosi pensoso* and *Nell'acqua chiara*, since a random distribution of surviving sources would predict a couple of pieces appearing in four manuscripts. To put it another way, the number of sources in which a work appears is significant only in relation to the total number of available sources in which it could have appeared.[15]

As a second example, Table 5 expands on a column of Table 3, to show how many liturgical Latin compositions survive in different numbers of copies, followed by a column showing the predicted number of pieces if all were equally popular, with a further column identifying the compositions which survive in five or more copies.[16]

The predicted number of pieces differs from the number of pieces we actually possess in two notable respects. First, the model predicts slightly fewer *unica*. This lower number is to be expected if some pieces actually were more popular than others, for, since the total numbers of copies of all the works is fixed, each concordance of a popular piece means one fewer concordance of a less popular work—thus more popular works also means more *unica*.

The second significant difference is a consequence of the first. Though the number of pieces in three or four sources fits the model, there are more pieces with five or more copies than the arithmetical model predicts. The prediction is that in only two out of every hundred simulations should there be even a single piece with six or more sources; instead there are four such pieces. These four are the few definitively popular Mass movements of the Papal Schism. The two pieces with five sources were probably, but not definitely, also popular. (Further discussion of popularity and these definitions appears in the appendix.)

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15. Although it is difficult to say for sure which pieces were definitely popular, that does not excuse the injustices done by the lack of performances of many works which survive in four, five, or more sources. David Fallows in 1975 drew attention to a neglect of Bartolino da Padova on disc, since only slightly ameliorated ("Performing Early Music," 252–53, 260). He reminded us that if we use the number of surviving sources as "any yardstick of respect in the 14th century, Bartolino is especially important, for three of the ten most widely distributed Trecento pieces are by him." It should be noted though that one of these three works, *Imperial sedendo*, may not be by Bartolino at all, but by the otherwise unknown Dactalus de Padua, to whom it is attributed in *ModA*. In fact the added suffix, "fecit" (to my knowledge never again used in this manuscript), could be read as a reaffirmation of authorship in the face of a known conflicting attribution, as in: "Yes, Dactalus, and not someone else, composed this."

16. There is a slight difference between this chart and the previous because a sixth source for Zachara's *Gloria, "Micinella,"* in *London 82959*, was discovered after the 2004 cutoff for Table 2. I have adjusted this table to include the work among the items with six sources.

Table 5. Count of Liturgical Pieces Compared to the Predicted Number.

No. of Mss appearing in	Actual No. of pieces	Predicted No. of pieces	Titles and composers of surviving pieces
Seven	2	0.00	Zachara: Two Credos *PMFC* 13, 21 & 23
Six	2	0.03	Zachara: Gloria *Laus, Honor;* Zachara: Gloria *Micinella*
Five	2	0.24	Ciconia: Gloria *Suscipe Trinitas,* Engardus: Gloria *PMFC* 12, 7
Four	1	1.6	
Three	5	8	
Two	15	30	
One	85	71	

Other genres of Trecento music can be analyzed similarly. Table 6 lists the ten pieces that the analysis argues were definitely popular, along with five works that were probably or possibly popular in their time.[17]

Francesco's *Donna s'i't'ò fallito* stands out on Table 6 for appearing in so many sources (eight, not including a lauda contrafactum and a citation by Prodenzani). It is nearly impossible to believe that it was not a popular work for scribes to copy ca.1400.[18] That three of the four most popular ballate are by Francesco should not be too surprising—his popularity has never been seriously challenged in the literature. Between the madrigals and the cacce a wider variety of composers is represented (among them, Jacopo and Bartolino), but there are still no real surprises. The most significant surprise comes when considering the most popular sacred works of the Trecento. Intriguingly, all of them are by a single composer, Antonio Zachara da Teramo.

It is natural to want to ask why these works, particularly Zachara's, were popular; though it is impossible to be certain, it is worth speculating about the reasons. Zachara's Mass music seems to have a connection, especially at the beginning of works, with more simple polyphonic forms. These simpler forms had a wide distribution throughout Italy. For instance, the influence of homophonic mensural polyphony appears in Zachara's popular Gloria, "*Micinella.*" The opening is similar to the homophonic or nearly-homophonic mensural mass movements (rhythmicized *cantus planus*

17. New sources for Zachara's Credo *PMFC* 13, 23, Ciconia's Gloria *Suscipe Trinitas*, and Engardus's Gloria, *PMFC* 12, 7 are found in Cuthbert, "Trecento Fragments," 238, 173–79, and 252, respectively.

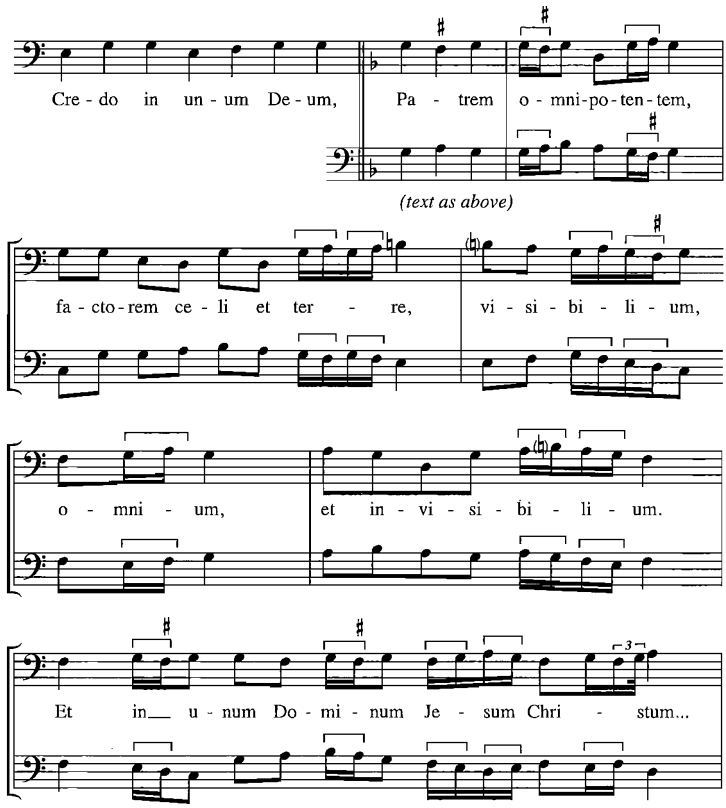
18. There are two released recordings of the work, Binkley and the Studio der frühen Musik's 1972 edition and Mary Springfels's Newberry Consort recording of 1990. See Morsanuto, "Discografia," 564, 581.

Table 6. Popular and possibly popular works (with source lists for liturgical works).

Liturgical: Undeniably Popular (7 Sources)	
Credo, *PMFC* 13, 21 (Zachara)	Bologna Q15, Boverio, Grottaferrata/Dartmouth, Pad D, ModA, Valladolid 7, Warsaw 378,
Credo, *PMFC* 13, 23 (Zachara)	Boverio, Cividale 98, Grottaferrata/Dartmouth, Kras., Siena 207, Trent 1563, Warsaw 378
Liturgical: Popular (6 Sources)	
Gloria: *Laus, Honor* (Zachara)	Bologna Q15, Munich Emmeram, Old Hall, Pad D, Siena 207, Warsaw 378
Gloria *"Micinella"* (Zachara)	Atri 17, Bologna Q 1, Bologna Q15, Bologna 2216, Grottaferrata/Dartmouth, London 82959
Liturgical: Possibly popular (5 Sources)	
Gloria: *Suscipe, Trinitas* (Ciconia)	Grottaferrata s.s., Grottaferrata/Dartmouth, Oxford 56, Pad D, Warsaw 378
Gloria, *PMFC* 12, 7 (Engardus)	Grottaferrata/Dartmouth, ModA, Pad D, Udine 22, Kras.
Caccia: Popular (6 Sources)	
Usellet(t)o selvag(g)io (Jacopo da Bologna)	
Caccia: Possibly popular (5 Sources)	
Tosto che alba (Gherardello)	
Madrigal: Popular (8 Sources)	
La douce çere (Bartolino da Padova)	
La bella stella (Giovanni da Cascia)	
O cieco mondo (Jacopo)	
Madrigals: Probably popular (7 Sources)	
O dolce appres'un bel pelaro (Jacopo)	
Ballata: Undeniably popular (8 Sources)	
Donna s'i't'ò fallito (Francesco da Firenze [=Landini])	
Ballate: Popular (7 Sources)	
Con langreme bagnandome (Johannes Ciconia)	
Gentil aspetto (Francesco)	
S'i'ti so(n) stato (Francesco)	

binatim) which flourished during the late Trecento and early Quattrocento. Example 1 shows a representative example, a Credo setting from *Vatican* 657.[19] This movement is perfectly homophonic for the first two lines of music and nearly perfect throughout. The phrases have a tendency to use longer note-values at the beginning and ends, and semibreves and minims

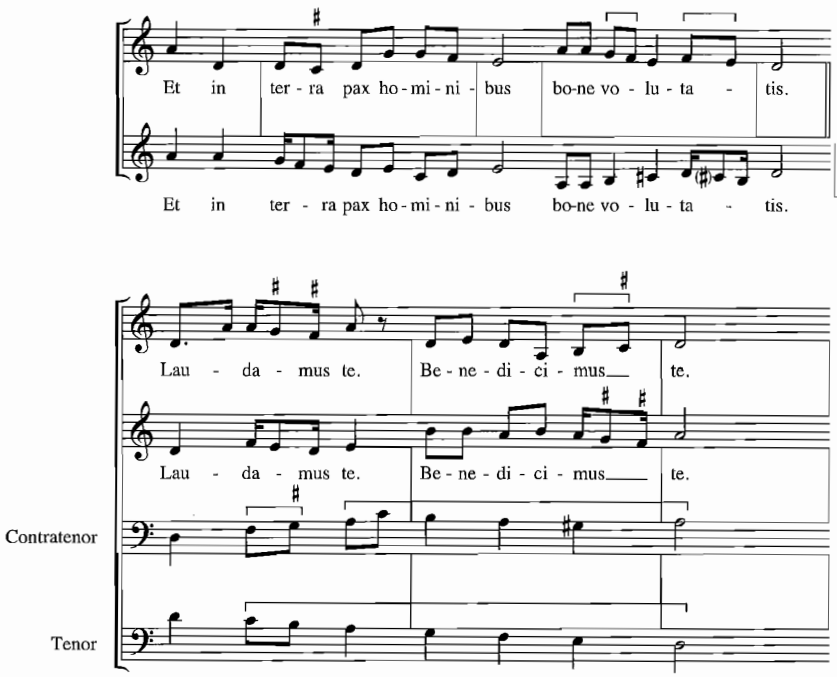
19. Example 1 is also edited in Fischer and Gallo, *PMFC* 12, 45.



Example 1. Opening of a homophonic Credo from *Vatican 657* (16:1 reduction. The notation of this example follows that of the source.)

in the middle and before cadences.[20] Some pieces of homophonic polyphony, such as the Credo *Cardinalis* even accelerate from their opening longs, through breves, to semibreves, and finally minims before allowing the note-values to occur in any order.

20. This tendency may be related to the Trecento practice of placing long melismas with faster notes on the penultimate syllable of a phrase. However, unlike their use in secular genres such as the ballata or more particularly the madrigal, in sacred works the shorter note-values begin several syllables before the cadence.



Example 2. Zachara, Gloria "*Micinella*," opening (8:1 reduction).

The Gloria "*Micinella*" of Zachara also begins homophonically, in two voices, which is almost a trademark opening for Zachara's Glorias.[21] Where the two voices are not homophonic, they introduce a short syncopated figure in the upper voice, as on "terra" in Example 2. The use of this rhythmic figure in an otherwise homophonic texture is typical of simple mensural polyphony.[22] (The Gloria found in *Warsaw 378* is similar, in that

21. This edition follows the lead of the three fragmentary sources of the Gloria in reversing the ordering of the cantus 1 and cantus 2 openings when compared with the order found in *Bologna Q15*. The voice-part cut away below the cantus 2 in *Bologna Q1* can definitively be identified as the contratenor on the basis of 19 surviving minim and ligature *cum opposita proprietate* stems, of which 18 line up with the stems in *Q15*'s contratenor after digitally lining up the two sources in Photoshop.

22. See, for instance, the hymn, *Iste confessor* in *Cividale 57*, *PMFC* 13, 39 (also transmitted with nearly the same outer voices, but different inner voices in *Apt 16bis*). The outer voices break homophonic rhythm in only two places, once with the figure Zachara uses; concordances of this work strongly suggest that only the two homophonic voices are original. The Credo in *Parma 09*, ff. Q–U provides an example of the same phenomenon in augmented values.

it also recalls rhythmicized, homophonic *cantus binatim:* this is a characteristic that helps argue for Zachara's authorship of that piece.)

In other popular works, Zachara puts different simple forms at the beginning. In both "Factorem" Credos (*PMFC* 13, 21 and 23), he continues the plainchant beyond its traditional ending at "Credo in unum deum" by setting "Patrem omnipotentem" monophonically as well. Credo 13, 21 uses the Credo I formula, a formula that was gaining in popularity at the end of the century—it is also used for Zachara's Credo "du village," the first of many settings by later composers—and Zachara could have been counting on the listeners to recognize this formula ("knowing an old warhorse") before jolting them with something both original and at a much increased rhythmic pace. Significantly, *ModA*'s decorated version of the piece includes no decorations at the opening, as if they were being held in reserve for after the suspense had been lifted.

Counting our Losses and Reflecting on the Surviving Repertory

The rate in which fragments have been found has increased rather than declined over the last forty years, and there is no reason to expect that the rate will drop off in the near future.[23] But new manuscript finds, especially of secular music, are more likely to result in new concordances than in new pieces. Given these realities, how we approach the surviving sources and what we learn from concordant readings and fragments becomes increasingly important to the development of the field.

When we suspect a single source or small group of sources of being representative of a much larger lost collection of music we should be inclined to grant that source or group more weight in our analyses. The monophonic instrumental compositions in the London codex (*29987*) are examples of pieces to which we apply further weight and study because they are presumed to stand in for a much larger repertory.[24] The *Rossi Codex* and the new *Mischiati Fragment* are other examples that stand in for a presumably larger

23. For evidence of the increasing rate of the discovery of sources, see Cuthbert, "Trecento Fragments," 10–19. A more mathematical treatment of the main arguments of this paper appears in the same dissertation on pages 44–86.

24. In addition to *London 29987*, the *Faenza* codex, as the only surviving large collection of Trecento keyboard music, has stood in for a larger group. On Faenza's keyboard repertories, see Plamenac, "Keyboard Music," Carapetyan, *Italian Source*, Memelsdorff, "Motti a motti," and *idem*, "Filiation and Transmission." Cattin, "Ricercheche sulla musica" discusses keyboard music of *Padua 553*. A manuscript of assuredly instrumental music notation written

lost repertory of music from mid-century and earlier. A single source that represents many missing pieces should carry more force in preparing descriptions of the typical music of a period than sources from repertories that are more complete. Thus, collections of sacred music, a repertory that survives at a lower rate than secular composition, need to take a more prominent position in descriptions of the late Italian *Ars Nova* as a whole.

A more prominent position in scholarship could also be accorded fragmentary sources in general. Sources are too often viewed according to their extant state rather than by what can be inferred about their original states. The majority of fragmentary manuscripts were originally similar in length to those few sources which do survive in complete or mostly complete state. Our losses are represented by the disembodied folio numbers that stand in for so many lost pages (Table 7).[25]

These numbers do not represent the original length of these manuscripts, but merely the highest numbered folio that currently survives. There are sometimes signs of even greater length. For instance, although the highest folio number on *Pad A* is 50, its gathering structure shows that the original manuscript contained at least 70 folios. Another hint at the original length of a manuscript is given by the order of works in *Florence 5*. The fragment presents a mainly alphabetical selection of Francesco da Firenze's ballate beginning with the letter "B" then "C" (*Che pen'è quest'al cor, Cholgli ochi assai ne miro,* and *Cosa nulla*). If this ordering continued to the end of

in score, *Assisi 187*, was discovered and described in Ziino, "Antico 'Kyrie'." Along with the single pieces of *Padua 553* and *Assisi 187*, many of the pieces in *Faenza* are sacred; thus the single-voice instrumental dances of *London 29987* take on even greater importance for their uniqueness. The traces of improvised polyphony within the art-song repertoire have also been seen as representatives of a lost tradition. This unwritten tradition has been discussed in Pirrotta, "New Glimpses," and Stone, "Glimpses of the Unwritten."

25. Besseler reported seeing the folio number 242 on *Parma 75*, but I have found no trace of this number even under ultra-violet light (Besseler, "Studien I," 231). Even the number 233 is questionable; the "2" is so faint that it may simply be the number "33" preceded by a stray pen mark. Huck, "*Frammenti Musicali*," 78, discovered a foliation number on a previously unnoticed binding strip of *Perugia Cialini*. Huck read the number as *clxxi*, but my own study of the fragment in Perugia suggests *clxvi* instead. Since Brumana and Ciliberti, *Frammenti Musicali* did not notice this folio number, their highest identified foliation is 36. An arabic folio number of 217 appears on *Frosinone 266/267*, but I do not believe that this foliation number is original and have therefore used the roman foliation. If we extended the period of this paper's study somewhat, the results would not change. Among slightly later manuscripts, we find the foliation number 125 on the *Boorman Fragment*. Among slightly earlier polyphonic sources, *Venice Giorgio Maggiore* contained at least 86 folios.

Table 7. Surviving folio numbers on Trecento fragmentary manuscripts.

Parma 75	233
Perugia Cialini	166
Frosinone 266/267	133
Stresa 14	141
Florence 5	138 (?)
Ciliberti Fragment	97
Todi Carità	93 (?)
Brescia 5	71
Siena Ravi 3	70
Vatican 1969	60
Pad A	50
Munich 3223	22
Florence Conservatorio	19

the alphabet, Francesco's section alone would need at least thirty folios, if not more, to complete it.[26]

Surviving folio numbers also convey a sense of the average length of a Trecento musical manuscript. As tempting as it might be to suppose that manuscripts were usually dismembered from their extremes, there is little evidence for any consistent order in their destruction. Many fragments, such as *Florence 5* (just discussed), are incomplete at both ends and imply further leaves that once existed on either side. If there is no better way of predicting the loss of specific folios in fragmentary manuscripts than pure random chance, then simple algebra can estimate the length of the typical manuscript. That length would be twice the average of the surviving folio numbers.[27] Since the average of the entries on Table 7 is 99, the average fragmentary manuscript is predicted to originally have had 198 folios. For comparison, the surviving intact Florentine codices average 171 folios.

26. The estimate was created by seeing that of Francesco's thirteen known ballate with alphabetically-ordered incipits between *Benché ora* and *Cosa nulla*, *Florence 5* provides readings for six, or just under 50%. Given that the scribe fits five (mostly two-voice) ballate on two folios, we can calculate the number of folios it would take to fit half of Francesco's 155 pieces as at least 30.

27. The formula for the expected length of a manuscript is derived in Cuthbert, "Trecento Fragments," 48. We can use the highest surviving folio number (Table 7) to compute the expected length of the original manuscript only when the number of folios that survive is few with respect to the original number. This is the case for all the fragments discussed above.

These two numbers are so similar that there is little reason to believe that most fragments were any different in their original length from those manuscripts that remain intact.[28]

* * *

The revelation is that we are not seeing only the tip of the iceberg of late-medieval Italian music, and this should embolden scholars to ask similar questions about other early repertories. If we turn for a moment to one piece of evidence from the late fourteenth-century French orbit, we see that Italy is not unique. The most useful document for ascertaining the extent of our losses in French music is the index page of a lost manuscript of motets and other works formerly in the possession of the Duchess of *Trémoïlle*. Though the loss of this manuscript itself is a misfortune for musicology, it is comforting to know that most of its pieces survive in other sources. Of its 114 compositions, between 74 and 78 (65–68%) survive, and this falls right in line with the pattern for Italian compositions.[29]

Further uses of models of popularity and loss hold great promise for musicology and other areas of research in the humanities. In musicology, the total number of chants sung in a region or the number of folk songs recalled by a group of people could be estimated. We could figure out the probability that a Renaissance motet that is unattributed in many sources was unattributed as a result of chance. Beyond musicology, the methods could be refined and reapplied to answer questions in other domains. Numismatists might be interested in the number of different types of coins in circulation at a certain time and in a given region. Codicologists could have a better understanding

28. The surviving Florentine manuscripts have the following lengths: *Panciatichi:* 115; *Pit.:* 150; *London 29987:* 185 (palimpsest numbering); *San Lorenzo 2211:* 188 (highest surviving folio); and *Squarcialupi:* 216.

29. The most recent inventory of the manuscript is Bent, "Note on the Dating." Staehelin, "Mehrstimmige Repertoires," 156 records 72 concordant works: he probably missed the concordance to *Se Lancelos* reported in Strohm, "Ars Nova Fragments," 115, and Peter Lefferts's later identification of a concordance for *Plausu querulo* in *Cortona 1*, published in Di Bacco and Nádas, "Papal Chapels," 86. Staehelin's article concentrated on losses of sources rather than of works and, as such, focused on library catalogs, payment records, and assumed omissions in stemma as his most important evidence. Two pieces in *Trémoïlle* have possible concordances that are tentative on account of their late dates (*De touts les biens* and Zachara's *Sumite Karissimi*) and two further Mass movements do not provide enough information to identify concordances. These four pieces account for the three percent uncertainty in the calculations above.

of the economics and chronologies of papermaking if they possessed more accurate estimates of the total number of watermark types originally produced in a particular region and time.

This examination of the Italian musical sources of the Great Papal Schism should remove a weight of insecurity from late-medieval musicology. Scholars now need have little worry that they are viewing only a small, possibly unrepresentative sliver of the original written repertory. Whether derived through statistical models or through literary evidence, the conclusion remains the same: between half and three-quarters of the written music of late-Trecento and early-Quattrocento Italy is available for study, most in modern editions. The most popular works of the time can be identified and await closer analysis. Finally, these techniques need to be applied more generally, to remove unfounded assumptions about other repertories and other eras. The use of estimates of survival and loss in medieval musicology and in the humanities in general is in its earliest infancy. Many more applications and results surely remain unseen and undiscovered, hidden just out of sight.

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Manuscript Sigla

Apt 16bis	Apt, Cathédrale Sainte-Anne, Bibliothèque du chapitre, Trésor MS 16*bis*.
Assisi 187	Assisi, Biblioteca Comunale, MS 187. (Currently housed in the B. Sacro Convento.)
Atri 17	Atri, Museo della Basilica Cattedrale, Biblioteca del Capitolo della Cattedrale, Frammento 17 (*olim* Archivio Capitolare. Sala Innocenzo IV, Cartella A, frammento 5).
Bologna Archivio Covers	Bologna, Archivio di Stato, Notarial covers of documents from 1337, 1338, 1369, 1412–13, and 1444.
Bologna Q1	Bologna, Museo Internazionale e Biblioteca della Musica di Bologna (*olim* Civico Museo Bibliografico Musicale), MS Q1, frammento n. 23 (*olim* 12).
Bologna Q15	Bologna, Museo Internazionale e Biblioteca della Musica di Bologna (*olim* Civico Museo Bibliografico Musicale), MS Q15 (*olim* Liceo 37). (BL)
Boorman Fragment	New York, Personal library of Stanley Boorman. (Boo)
Boverio	Turin, Biblioteca Nazionale Universitaria, T.III.2. (Turin 2)
Brescia 5	Brescia, Biblioteca Civica Queriniana, Flyleaves in Inc. C.VI.5. (Quer)
Ciliberti Fragment	Perugia, Personal library of Biancamaria Brumana and Galliano Ciliberti. (Cil)
Cividale 57	Cividale del Friuli, Museo Archeologico Nazionale, MS LVII.
Cividale 98	Cividale del Friuli, Museo Archeologico Nazionale, MS XCVIII. (from Cividale A)
Cortona 1	Cortona, Archivio Storico del Comune, Fragment without shelfmark in a white folder marked only as "Comune" [fragment 1].
Faenza	Faenza, Biblioteca Comunale, MS 117. (Fa)
Florence 5	Florence, Biblioteca Nazionale Centrale, Incunab. F.5.5.
Florence Conservatorio	Florence, Biblioteca del Conservatorio di Musica, "Luigi Cherubini," Cassa forte 74 (*olim* D 1175). (Florence 1175, FC)

Frosinone 266 Frosinone, Archivio di Stato, Collezione delle pergamene 266 (31).

Frosinone 267 Frosinone, Archivio di Stato, Collezione delle pergamene 267 (38).

Grottaferrata s.s. Grottaferrata, Biblioteca del Monumento Nazionale (within the Abbazia Greca di S. Nilo), MS without shelfmark, in a light gray binder currently on the same shelf as Lat. 224.

Grottaferrata/Dartmouth Grottaferrata, Biblioteca del Monumento Nazionale (within the Abbazia Greca di S. Nilo), [Crypt.] Lat. 224 (*olim* Collocazione provvisoria 197) (Grot, GR 197, GR) and Hanover, New Hampshire, Dartmouth College Library, MS 002387 (*olim* Santa Barbara, Accademia Monteverdiana, fragment without shelfmark).

Houghton 420 Cambridge (Massachusetts), Harvard University, Houghton Library, MS Lat. 420.

Kras. Warsaw, Biblioteka Narodowa, MS III. 8054 (*olim* Biblioteka Świdzińskich, then Biblioteka Krasiński, then Biblioteka Narodowa, 52).

London 29987 London, British Library, Reference Division, Department of Manuscripts, Additional MS 29987. (Lo, L, LB)

London 82959 London, British Library, Reference Division, Department of Manuscripts, Additional MS 82959.

Manganelli Fragment Fiesole, Personal library of Michele Manganelli, Inv. 50.

Mischiati Fragment Reggio Emilia, Archivio di Stato, Archivio Comune Re, Appendice, Frammenti di codici musicali (no. 16).

ModA Modena, Biblioteca Estense e Universitaria, MS ∝.M.5.24 (*olim* IV.D.5, then lat. 568).

Munich 3223 Munich, Bayerische Staatsbibliothek, Musiksammlung. MS mus. 3223. (MüK)

Munich Emmeram Munich, Bayerische Staatsbibliothek, Handschriften-Inkunabelabteilung. MS lat. 14274 (Tresorhandschrift 1; *olim* mus. 3232a; Cim. 352c). (MuEm, Em)

Old Hall London, British Library, Reference Division, Department of Manuscripts, Additional MS 57950 (*olim* Old Hall, Library of St. Edmund's College, MS without shelfmark). (OH)

Oxford 56 Oxford, Bodleian Library, MS Canon. Pat. Latin [Scriptores Ecclesiastici] 56.

Pad A Oxford, Bodleian Library, MS Canon. Pat. Latin [Scriptores Ecclesiastici] 229 (Oxford 229) and Padua, Biblioteca Universitaria, MSS 684 and 1475 (Padua 684 and Padua 1475).

Pad D	Padua, Biblioteca Universitaria, MSS busta 2/1 (from MS 1283), busta 2/2 (from MS 1225), busta 2/3 (from MS 675), and MS 1106. (Padua 1283, Padua 1225, Padua 675, and Padua 1106).
Padua 553	Padua, Archivio di Stato, Fondo Corporazioni soppresse, S. Giustina 553. (Small book in a special folder called registro membranaceo/libro n. 4).
Panciatichi	Florence, Biblioteca Nazionale Centrale, Panciatichiano 26. (Pan, FP, FN, Fl, F)
Parma 9	Parma, Archivio della Fabbrica del Duomo. F 09.
Parma 75	Parma, Archivio di Stato, Raccolta Manoscritti, busta 75, n. 26 (*olim* Armadio B, Busta n. 75, fasc. 2) ex convento LXXXV (S. Servino di Piacenza) reg. n. 52.
Perugia Cialini	Perugia, Biblioteca del Dottorato dell'Università degli Studi, Incunabolo inv. 15755 N.F.
Pit.	Paris, Bibliothèque Nationale de France, fonds italien 568 (*olim* Bibliothèque Royale 165 du Supplément, then Nouv. Supplément Fr. 535) (P, It).
Reina	Paris, Bibliothèque Nationale de France, fonds nouvelles acquisitions françaises 6771. (R, PR, Rei)
Rossi	Rome, Biblioteca Apostolica Vaticana, Rossianus 215 and Ostiglia, Opera Pia Greggiati, Mus. rari B 35 (*olim* MS without shelfmark). (Rs, R)
San Lorenzo 2211	Florence, Biblioteca Medicea-Laurenziana, Archivio Capitolare di San Lorenzo, MS 2211.
Seville 25	Seville, Biblioteca Colombina de la Institución Colombina, MS 5.2.25 (*olim* Z Tab. 135, n. 32, then BB-147–32) (Sev)
Siena 207	Siena, Archivio di Stato, Frammenti Musicali busta n. 1. inserto n. 11 (*olim* Frammenti di musiche, n. 207; Also with separate catalogue numbers 326 (=bifolio) and 327 (=folio), and Mostra Vetrina n. 40).
Siena Ravi 3	Siena, Archivio di Stato, Frammento sulla copertina del registro "Ravi 3" dal Fondo Vicariato di Gavorrano (1568–69).
Squarcialupi	Florence, Biblioteca Medicea-Laurenziana, Mediceo Palatino 87. (Sq, FL)
Stresa 14	Stresa, Biblioteca Rosminiana, Collegio Rosmini al Monte, MS 14 (*olim* Domodossola, Convento di Monte Calvario). (Str, Dom)

Todi Carità Todi, Archivio Storico Comunale, fondo Congregazione di Carità, Istituto dei sartori, Statuto [senza segnatura] (ex O. p. Sarti n. 83).

Trémoïlle Paris, Bibliothèque Nationale, fonds nouvelles acquisitions françaises 23190 (*olim* Angers, Château de Serrant, Duchesse de la Trémoïlle). (Trém)

Trent 60 Trent, Fondazione Biblioteca di S. Bernadino (*olim* dei Padri Francescani), Incunabulo n. 60.

Trent 1563 Trent, Museo Provinciale d'Arte, Castello del Buon Consiglio, MS 1563 (Manuscript belonging to the Biblioteca comunale).

Turin 9 Turin, Biblioteca Nazionale Universitaria, J.II.9. (TuB)

Udine 22 Udine, Archivio di Stato, Frammento 22 (*olim* Arch. Not. Antico, busta 773). (from Cividale A)

Valladolid 7 Valladolid, Archivo de la Real Chancillería, Pergamino, carpeta 29, documento 7.

Vatican 657 Rome, Biblioteca Apostolica Vaticana, Barberinianus latinus 657.

Vatican 1969 Rome, Biblioteca Apostolica Vaticana, Ottobonianus latinus 1969.

Venice Giorgio Maggiore Venice, Monastero di San Giorgio Maggiore, Fragment without shelfmark.

Warsaw 378 Warsaw, Biblioteka Narodowa, Lat. F. I. 378 (*olim* St. Petersburg, Imperial Public Library, same call number). (Manuscript lost.) (StP)

Appendix
Statistical Models

Some Probability Basics

A probability is defined as a number between 0 and 1 (inclusive), and represents the likelihood of an event happening. For example, if we roll a fair six-sided die, the chance that we get a five is 1 in 6. That is, there are six possible outcomes, of which one gives us the sought after outcome. We can write that a is the event "roll 5" and $\Pr(a) = 1/6$.

The probability of something *not* happening is defined as one minus the probability of the event happening. So Pr(roll something other than 5) $= \Pr(a \text{ does not occur}) = 1 - 1/6 = 5/6$.

If x and y are independent events, like dice rolls or scribes working on unrelated manuscripts, then the probability of x and y both happening is $\Pr(x \text{ and } y) = \Pr(x) * \Pr(y)$.

In addition to knowing how likely it is that something will occur (probability) we also often want to know how many times an event will occur if we keep performing or observing a certain action. For instance, if we go back to the example of dice, you may want to know how many times you would *expect* to roll a five if you rolled a die ten times. We call this rational expectation the *expected value* (EV).

Fortunately, for independent events, such as dice rolls, in which what you rolled previously does not affect what you are likely to roll next, all one needs to do to calculate expected value is to multiply the probability of an outcome by how many times it is done.

So on average the expected number of fives after ten dice rolls is:

$$\text{EV} = \text{number of rolls} * \Pr(\text{roll a five}) = 10 * {}^{1}/_{6} = {}^{10}/_{6} \text{ or } 1.67$$

Of course, it is impossible to roll 1.67 fives. What it means is most often two of the ten rolls would be a five; sometimes it will be one, sometimes three; less often zero or four, and almost never nine or ten.

Applications to Estimating the Numbers of Lost Pieces of Music

Most of the expressions derived below will depend on n—the total number of pieces that once existed. That is to say, the first equations will depend on

the value we are ultimately trying to estimate. What we will do is find a way to relate the number n to the number of pieces that survive today. Then we can try to find the most likely number for n that gives an answer that accords with the surviving number of compositions.

Let $\{x_1, x_2, \ldots, x_n\}$ be the set of pieces in a given genre that may have once existed. Any given piece, x, is either a work that exists today or one that is no longer extant. Both types of pieces are found in the set. Each piece x in the set is numbered from 1 to n.

Let $\{m_1, m_2, \ldots, m_y\}$ be the set of manuscripts now surviving. We call k_i the number of pieces in manuscript m_i. Unlike $\{x_1, x_2, \ldots, x_n\}$ which represents all pieces once copied, this set only comprises manuscripts or fragments around today. The manuscripts in this set are numbered 1 to y, and thus y equals the total number of manuscripts with one or more pieces in a given genre. It is a known quantity (e.g., for madrigals, $y = 18$).

The compiler of manuscript m_1 chooses k_1 different pieces to place in it. There are any number of reasons why the person writing the manuscript might choose a given piece to be in the manuscript—the audience of the manuscript, the pieces known to the scribe, forms to be represented, etc.—but among the pieces in a single sub-genre, it can be difficult for us to tell why certain pieces are chosen or not.

We will begin by supposing that within each genre the pieces chosen are as good as random. This is an assumption that will raise some eyebrows; please bear with it for now, as later I will show that this assumption turns out not to be far off. Given this supposition of random collection, the probability than any piece appears in any manuscript m_1 depends just on the number of pieces in the manuscript and the total number of pieces (in this genre). The probability that any piece appears in a manuscript of k_1 pieces is simply k_1/n, where n again is the total number of pieces once copied—the unknown for which we search.[30] The probability therefore that any given piece does *not* appear in a manuscript is given by subtracting the previous formula from 1, that is: $(1-k_1/n)$.

30. This formula holds true for cases where the total number of pieces in any given manuscript is small compared to the size of the total repertory. If the manuscript is sufficiently large that we think it held a high percentage of all the pieces that ever existed, then some corrections must be made (and are made in the more mathematical version of the formulas presented in Cuthbert, "Trecento Fragments"). However, it should be noted that a skeptic who believes that some of our single manuscripts hold a large percentage of the pieces that once existed (and therefore that this paper must use the more complex formulas) cannot simultaneously believe that all our manuscripts combined represent a small fraction of the pieces that once existed.

For two manuscripts that were compiled independently of each other (excluding, for example, the Machaut manuscripts, but none of the principal Trecento manuscripts), the probability that a piece does not appear in either of the two manuscripts is given by multiplying the probability it does not appear in the first manuscript by the probability the same piece does not appear in the second. That is, the probability that x_1 does not appear in m_1 and also x_1 does not appear in m_2 is the product of the two terms:

$$\Pr(x_1 \text{ does not appear in } m_1) * \Pr(x_1 \text{ does not appear in } m_2) = \left(1-\frac{k_1}{n}\right)\left(1-\frac{k_2}{n}\right)$$

Elementarty algebra reduces the previous equation to $\left(\frac{n-k_1}{n}\right)\left(\frac{n-k_2}{n}\right)$ or more simply $\frac{(n-k_1)(n-k_2)}{n^2}$. We can then generalize this statement to find the probability of x not appearing in any extant manuscript:

$$\Pr(x_1 \text{ does not appear in } \textit{any} \text{ MS}) = \left(1-\frac{k_1}{n}\right)\left(1-\frac{k_2}{n}\right)\cdots\left(1-\frac{k_y}{n}\right)$$

$$= \left(\frac{n-k_1}{n}\right)\left(\frac{n-k_2}{n}\right)\cdots\left(\frac{n-k_y}{n}\right) = \frac{(n-k_1)(n-k_2)\cdots(n-k_y)}{n^y}.$$

The principle of expected value (discussed above) can then be brought in to estimate how many pieces are expected to be missing today, given the manuscripts that survive and the number of pieces there once were in the Trecento.

The expected value of the number of pieces not appearing in any MS that survives today is simply the probability that any given piece does not appear in any manuscript multiplied by the total number of pieces, our unknown n:

$$\text{EV (missing pieces)} = n * \Pr(x \text{ does not appear in } \textit{any} \text{ MS}) =$$

$$n * \frac{(n-k_1)(n-k_2)\cdots(n-k_y)}{n^y} \text{ or } \frac{(n-k_1)(n-k_2)\cdots(n-k_y)}{n^{y-1}}.$$

It looks as if there are two unknowns here: the expected number of missing pieces (EV) and the total number of pieces (missing or known), n. But what is the expected number of missing pieces? It is simply the number of pieces that were written originally (n) minus the number we currently have (let us call that number r).

$$\text{EV (missing pieces)} = n-r$$

We can substitute this result into the previous equation:

$$n - r = \frac{(n-k_1)(n-k_2)\cdots(n-k_y)}{n^{y-1}}$$

In this equation, r and k_1, k_2, ..., k_y are all numbers we know, so there is only one variable, n. Since we have a single algebraic equation with a single unknown, it should be solvable. However, solving for n in this equation is not easy when y is a number above three or four; and since y is the number of manuscripts containing pieces in a particular genre, y will be on the order of ten to thirty. As the last equation is too complicated to solve directly, reducing it would have required tricky math decades ago. However, the solution can be closely estimated in seconds through computer-assisted "trial and error." We rewrite the previous equation as:

$$n - r = \frac{(n-k_1)(n-k_2)\cdots(n-k_y)}{n^{y-1}} = 0$$

and then write a program to try various numbers of n (theoretically, from $r + 1$ to infinity, but from $r + 1$ to 200,000 is good enough) until it finds the n which comes closest to solving this equation.[31] (We are unlikely to find the exact solution since n can be a fraction rather than a whole number). Solutions to this equation for cacce, madrigals, ballate, liturgical, and other Latin works, are used in the last column of Table 4.

Testing the Model: Cross-Validation

An important quality in a model is its ability to be tested and stand up to such testing. One common way of testing the validity of an estimate is via a technique called holdout cross-validation. In cross-validation and similar techniques such as bootstrapping and jackknifing, a subset of the data is run through the theory to see how well it predicts the full set of data (by comparing its results to our actual numbers).

For instance, in cross-validating I removed either the fragmentary sources (including *San Lorenzo 2211*) or the complete sources (leaving in *San Lorenzo* and the fragments) and then predicted how many additional pieces would be found if we added back the number of folios in those sources

31. Such a program could be written in any modern programming language. Examples of programs written in the Perl language used to solve the equations in this section are given in Cuthbert, "Trecento Fragments," 81–86.

that were removed.[32] Without the fragments or *San Lorenzo,* for instance, we would have 159 madrigals in 314 copies. On the basis of this information, the first model predicted that there were originally 175 madrigals, and via cross-validation estimated that if we had 65 more copies of madrigals, six of them would be new, for a total of 165 madrigals (See Table 8). As can be seen, there actually are 166—an extremely close estimate. For liturgical music the cross-validated prediction was off only by three from our observed number, 109 instead of 112: also very close. Running the same model for ballate, the cross-validation predicts that there should be 385 ballate instead of the 409 we do have—not as close as the other two genres, but this is still not an error rate that suggests that missing icebergs exist; we may want to hold in mind that there could be 20–30% more missing ballate than predicted by the model of a scribe collecting basically randomly. Taken as a whole, the cross-validation tests suggest that the model described above should predict the number of lost pieces with a high degree of certainty, only slightly underestimating the actual number. The degree of accuracy of the cross-validation tests also suggests that the assumption needed for the previous model namely, that random chance as a good predictor for how music is chosen, is not a terrible assumption at all.

To test estimates above via a cross-validation technique, I used the method above to find a value for n on the basis of a subset of the data. A similar model is then used to find an expected number r_1 for the number of pieces we would expect to have if we had new manuscripts m_{y+1}, m_{y+2}, etc. The calculations are much easier than before, since we begin with an estimate for n. For a first approximation, the portion of the repertory that is missing, i.e., $(n-r)/n$, when multiplied by the number of new pages in all the new manuscripts m_{y+1}, m_{y+2}, gives us the number of new pieces we should expect to find (which when added to r gives r_1).[33]

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32. It is important that the works chosen to be removed for hold-out are chosen either randomly (not in the researchers' control) or if chosen in some other way, the process is not repeated until the desired result is achieved. In the ideal situation, multiple tests would be run on randomly chosen subsets of the complete corpus of manuscripts. However, this proved impossible given the way the data is stored in the database this paper uses.

33. The simple method of cross-validation described above is slightly inaccurate, because the portion of the repertory that is missing changes with each new find. A more accurate test comes from computing a new expected value for the missing pieces using the new manuscripts. If j is the number of new manuscripts we have added then:

$$\text{EV(new \# missing pieces)} = n * \frac{(n-k_1)(n-k_2)\cdots(n-k_y)\,(n-k_{y+1})(n-k_{y+2})\cdots(n-k_{y+j})}{n^{y+j}}$$

Table 8. Cross-Validation Results.

	no. pieces pre-c.v.	predicted survival	actually surviving today
Madrigals	159	165	166
Liturgical Pieces	91	109	112
Ballate	357	384	409

Calculating the Expected Number of Copies in a Random Distribution

Another way of testing to see how well our first supposition, that of equal probability of collecting, holds up is to run a "Monte Carlo" simulation of work distribution. One way of conducting a Monte Carlo simulation is to put slips of paper in a hat containing the names of all known pieces in a given genre. Added to the hat is a numbered slip of paper for every lost piece predicted by the previous model. So there are as many slips of paper as there are total predicted pieces. Then for each surviving manuscript we draw as many slips of paper as there are pieces in that genre in that manuscript. For instance, since *Boverio* contains sixteen liturgical works, we will draw sixteen slips. It should be obvious that each piece is equally likely to be drawn, and that no piece can appear in the same manuscript twice. We record what pieces appeared and then replace the slips into the hat. The slips are shuffled in the hat, and the process is repeated for each manuscript. (The "Monte Carlo" aspect of the simulation stems from the role that probability or luck plays in determining the outcome, as in a casino in Monte Carlo.)

After conducting the simulation, there exists a record of which and how many pieces were drawn multiple times, which were drawn once, and which were never drawn. This equal-probability situation can be compared to our real-world situation to see how well they accord. In order to reduce the role of luck, the whole set of draws could be done many times and the average of the simulated draws could be used instead.

Since performing this simulation even once, let alone hundreds of times, would be extremely time consuming (i.e., draw, record, replace,

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Since n is a constant, this equation can be evaluated simply. We can then subtract the new number of missing pieces from n to get the number of pieces expected in this situation, and can compare that number to the number of pieces actually observed. Although the more complicated method has been used in obtaining the cross-validation numbers this paper uses, the actual difference in results from the first method is slight.

Table 9. Output of Simulating 10,000 Random Draws

(a)		(b)	(c)	(d)		(e)	(f)
7	=>	0.00	(0.00%)	(0.00	=>	0.00%)	[28]
6	=>	0.03	(0.01%)	(0.03	=>	0.02%)	[282]
5	=>	0.24	(0.13%)	(0.27	=>	0.14%)	[2165]
4	=>	1.61	(0.84%)	(1.88	=>	0.99%)	[8200]
3	=>	8.29	(4.34%)	(10.18	=>	5.33%)	[10000]
2	=>	30.52	(15.98%)	(40.69	=>	21.30%)	[10000]
1	=>	71.25	(37.30%)	(111.94	=>	58.61%)	[10000]
0	=>	79.06	(41.39%)	(191.00	=>	100.00%)	[10000]

reshuffle, and repeat *ad nauseam*), the draws are generally simulated by computer programs. Table 9 shows the output from the program which ran the simulation 10,000 times, distributing the liturgical pieces into hypothetical manuscripts and fragments the same length as those that survive.

Column (a) gives the number of times a piece is copied (7, 6, 5, etc.). Column (b) shows how many pieces with that many copies appeared. So we can see towards the bottom that on average there were 30(*ish*) pieces with two copies, 71 that were unica, and 79 that did not survive in any source. Column (c) gives the number in (b) as a percentage of the total number of works. Columns (d) and (e) (e.g., 40.69 => 21.30%) give a running total of columns (b) and (c). For instance, on average, the simulation predicts in column (e) that the number of works with two or more copies will be 21% of all copied compositions, known and lost. Column (f) shows how many times out of 10,000 that the simulation produced at least one work copied the number of times in (a). So from column (f) row two, we see that only 282 times out of 10,000 did a piece appear in six sources. We can interpret this number to mean that for a given piece that appears in six sources, there is a (roughly) 2.8% chance that it is an run-of-a-mill piece that happened to survive more often than chance, and a 97.2% chance that it was a popular piece. We should thus believe that the piece was popular in the late Middle Ages. However, it is easily seen that a piece in four sources could very well be a piece of middling popularity whose survival in many sources is merely due to chance. These percentages were used to create the labels in Table 6 of "Undeniably popular" (~ 0.2% or less that its survival in so many sources is due to chance), "Popular" (~0.5–3%), "Probably popular" (5–10%), and "Possibly popular" (10–25%).

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